Is the Brexit Bifurcation Causing Chaos in the United Kingdom?

Brexit versus the pandemic as a source of economic risk

A rare controlled experiment for economic research

Economic Risk Caused by the Pandemic

The effects of Covid on the UK economy are driven by external stochastic shocks from the virus.

The resulting economic stochasticity reflects the consequences of the extrinsic shocks from outside the economy.

Is Brexit Increasing Economic Risk within the UK?

- Advocates had argued that Brexit would decrease economic risk within the UK.
- Unlike the pandemic, Brexit is not a sequence of external shocks.
- **A bifurcation** that imposes/and or removes constraints from the pre-Brexit EU economy.

Why Is Brexit Changing Economic Risk without a Source of External Shocks?

1. Change in subjective probabilities?
   - Weak, circular explanation.
2. Multiple correlated equilibria with sunspots.
   - Why would the Brexit bifurcation increase multiple equilibria or sunspot shocks?
3. Bifurcation to or from chaos.
4. Chaos both before and after bifurcation.
   - Change in fractal geometry of attractor set.

Mathematical Chaos Found in the US Economy

"Shilnikov Chaos, Low Interest Rates, and New Keynesian Macroeconomics"

By William Barnett, Giovanni Bella, Taniya Ghosh, Paulo Mattana, and Beatrice Venturi

Cause and Effect

Source: bifurcation caused by adjoining an interest rate feedback (“Taylor”) rule to the system’s dynamics with New Keynesian sticky prices.

Effect: downward drift in interest rates caused by fractal geometry of the Shilnikov attractor set.

Multiple Bifurcation Subsets Previously Found Both for the US and UK Economies

- **Normal**: Existence of bifurcation subsets of parameter space normal in system dynamics.

- **Consequences**: Need not be between stability and instability. Can separate different kinds of stability or different kinds of instability.

- **US economy**: Found in many macroeconomic models, including the simplest New Keynesian model with three linear equations. Lack of bifurcation stratification very rare in dynamics.

- **UK economy**: Found in continuous time Bergstrom-Nowman-Wymer Keynesian macroeconometric model.

Mathematical Properties of Chaos

- **1. “Stochasticity” having deterministic origins.**
- 2. Positive dominant Liapunov exponent.
- **3. Fractal “strange” attractor set.**
- 4. Fractional Hausdorf dimension.
- 5. Sensitivity to initial conditions.

Why Care?

The fractal geometry of the attractor set contains **all of the information about the unknown system** that generated the system (Takens theorem).

The resulting “**intrinsic stochasticity**” contains valuable information, which should not be filtered out as useless extrinsic noise.

Examples

WEATHER.

CLIMATE

BRAIN WAVES

NATURE VERSUS ENGINEERING.

Fashionable Research

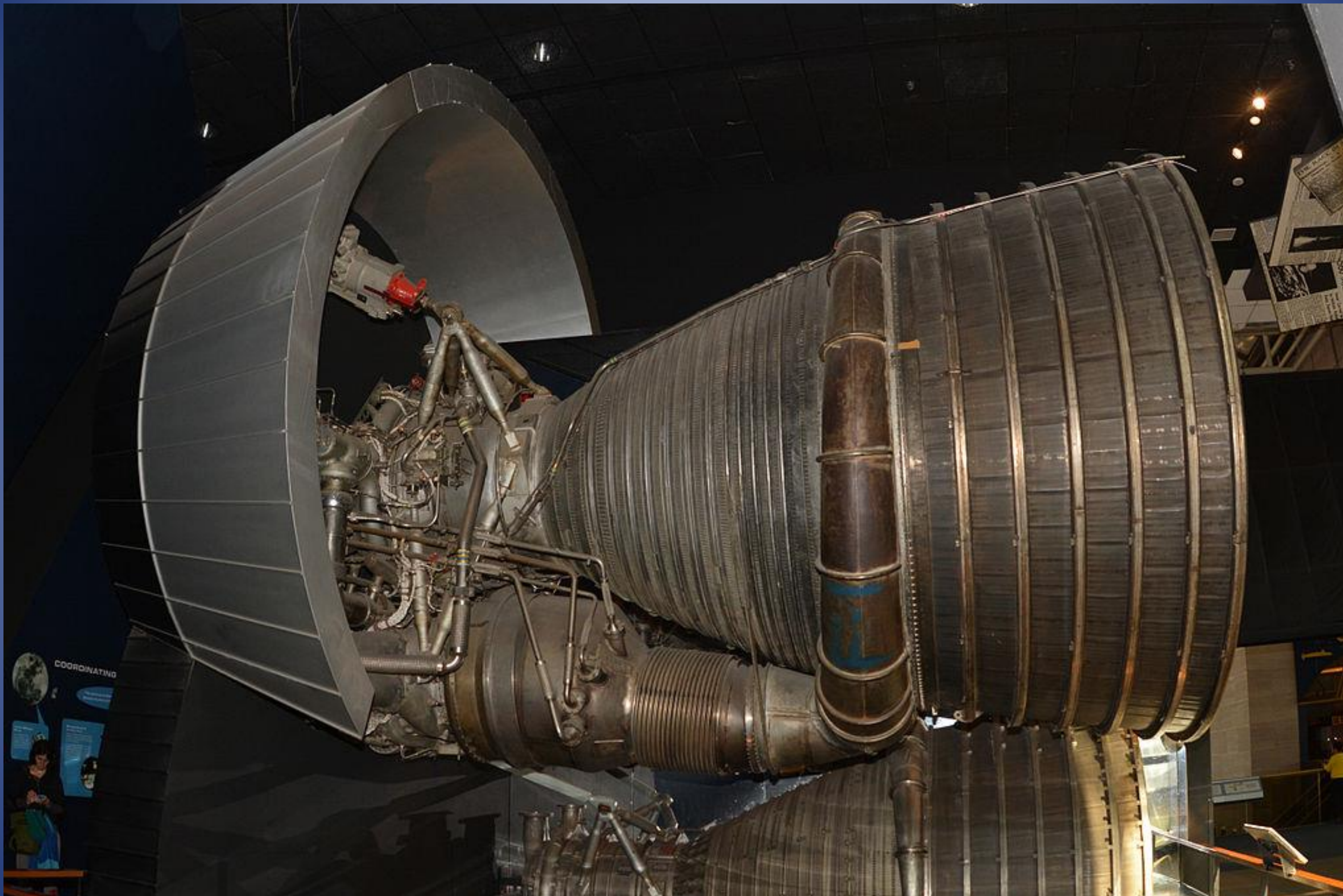
JURASSIC PARK MOVIE.

BEST SELLER POPULAR BOOKS.

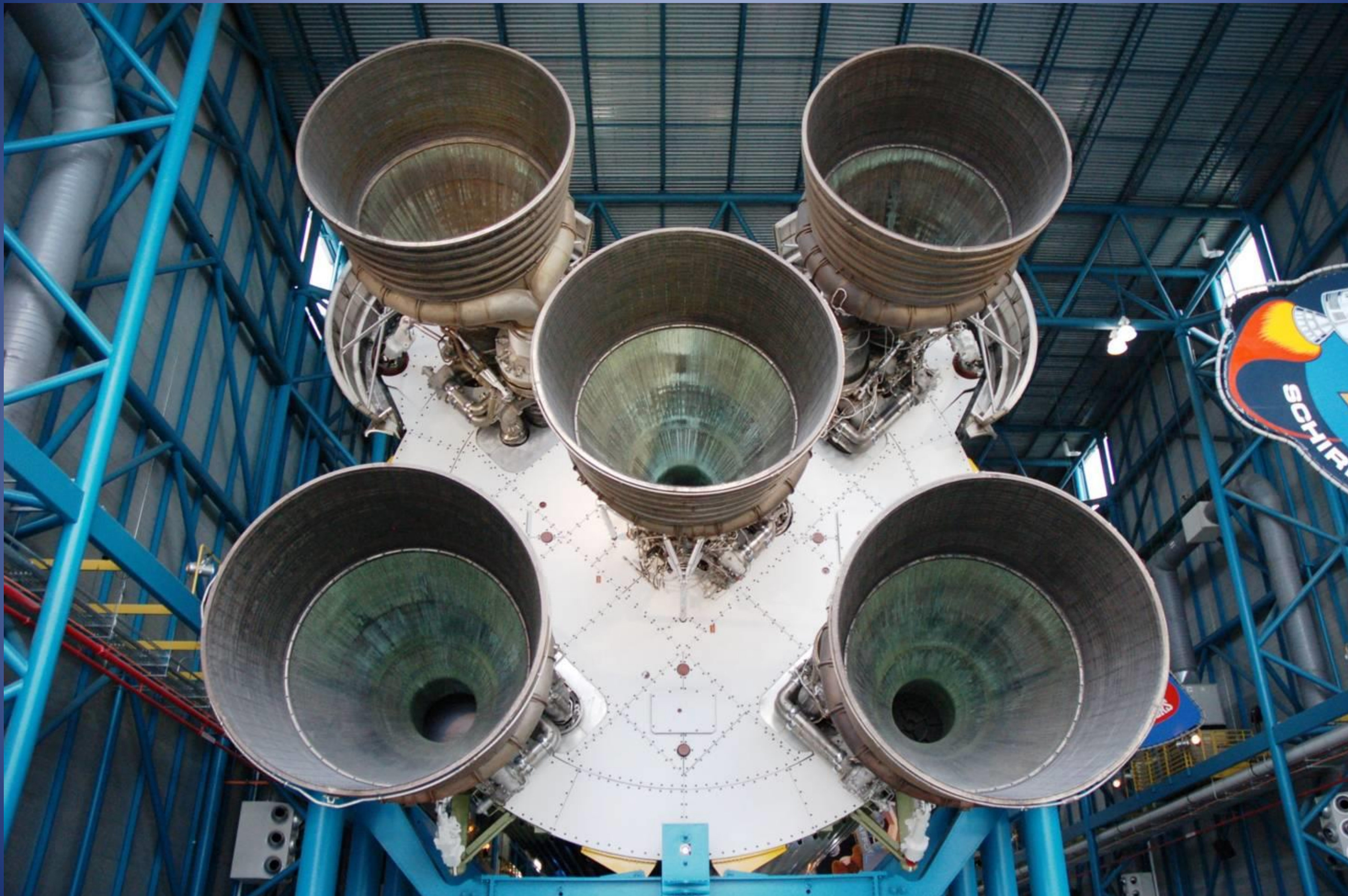
ARTICLES IN POPULAR MEDIA.

Systems Theory and Engineering

- Small changes in data or in parameters can cause large changes in dynamics.
  - Bifurcation and/or chaos.
  - Rocketdyne F-1 rocket engine.
    - Small change in parameter: explosion.


COORDINATING


VII




USA

Viscount Ilya Prigogine

- *Order Out of Chaos: Man's New Dialogue with Nature*, Bantam Books, 1984.
- "We grow in direct proportion to the amount of chaos we can sustain and dissipate."

Finding of Chaos in U.S. Economic Data

- William A. Barnett and Ping Chen,

  "The Aggregation Theoretic Monetary Aggregates are Chaotic and Have Strange Attractors: An Econometric Application of Mathematical Chaos,"

  in William A. Barnett, Ernst Berndt, and Halbert White (eds.), *Dynamic Econometric Modeling*, Cambridge University Press, 1988.

Economic Significance

Tests best used in laboratories with very large samples and controlled experiments.

With economics data, the tests have no way of telling whether the source of the chaos is from within the economy, or from entirely outside the world's economies, as from the weather.

Attempts to Condition upon Economic Structure, so can impute inferences to economy

- J. M. Grandmont (1985), *Econometrica*, classical model.
- A Bhaduri and DJ Harris, *QJE* (1987), "The Complex Dynamics of the Simple Ricardian System."
  (Donald Harris is Kamala's father)
  - All solutions are Pareto Optimal in both models.
- Papers with Yijun He on the UK Continuous Time Macroeconometric Model (Bergstrom et al).
  - Keynesian, so policy relevant.

Results with Continuous Time Model of the UK Economy

"Stability Analysis of Continuous Time Macroeconometric Systems," with Yijun He,

Studies in Nonlinear Dynamics and Econometrics, January 1999, vol 3, no. 4, pp. 169-188.

A. R. Bergstrom UK Model

- Bergstrom, A. R., K. B. Nowman and C. R. Wymer (1992), "Gaussian Estimation of a Second Order Continuous Time Macroeconometric Model of the UK," *Economic Modelling*, vol 9, pp. 313-351.

- Bergstrom, A. R. and K. B. Nowman (2006), *A Continuous Time Econometric Model of the United Kingdom with Stochastic Trends*, Cambridge U. Press, 2007

Model structure:

14 second order differential equations

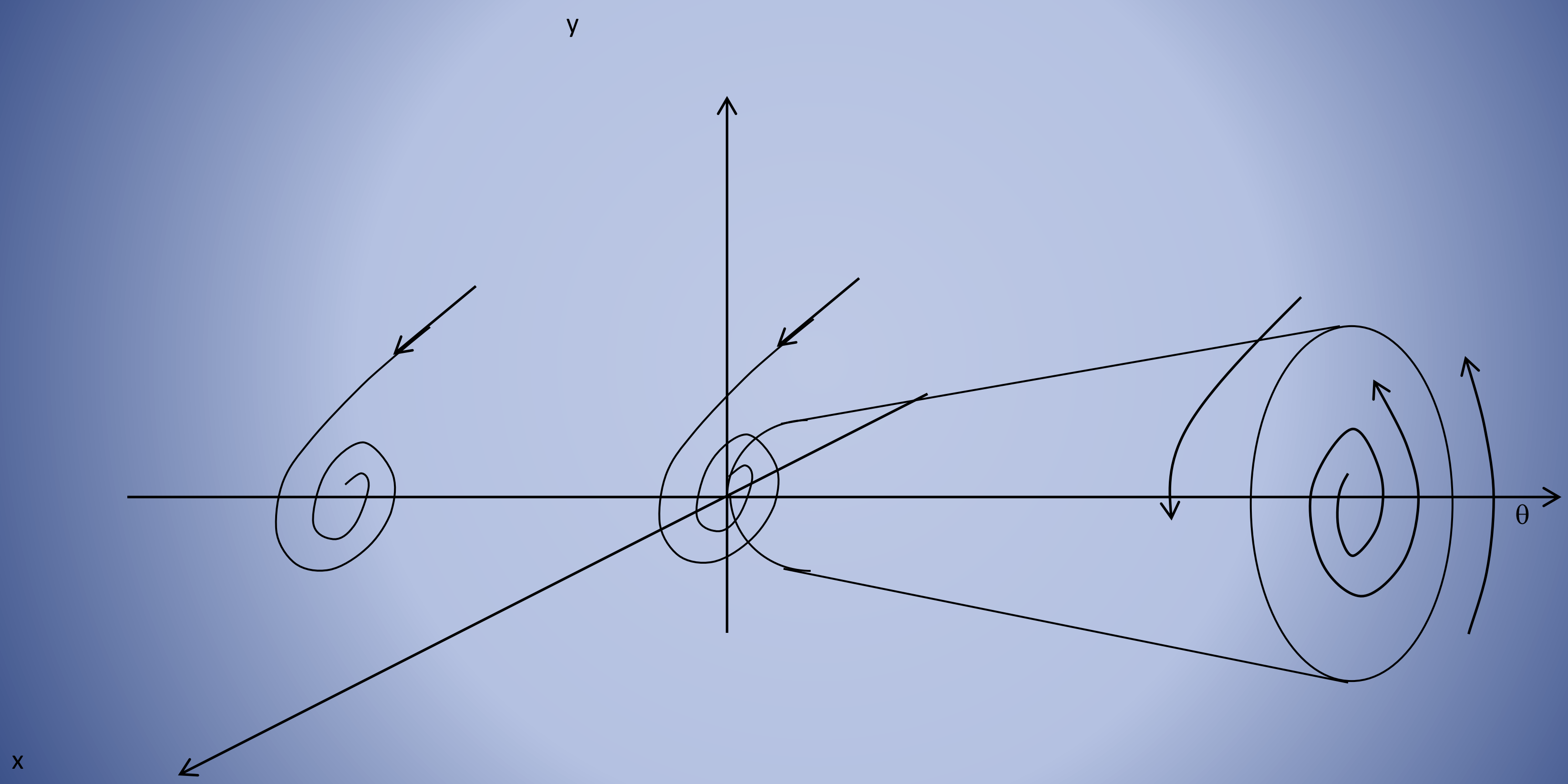
Structural parameters:

63 structural parameters, including 27 long-run propensities and elasticities and 33 speed of adjustment parameters

Free parameters:

3 trend parameters

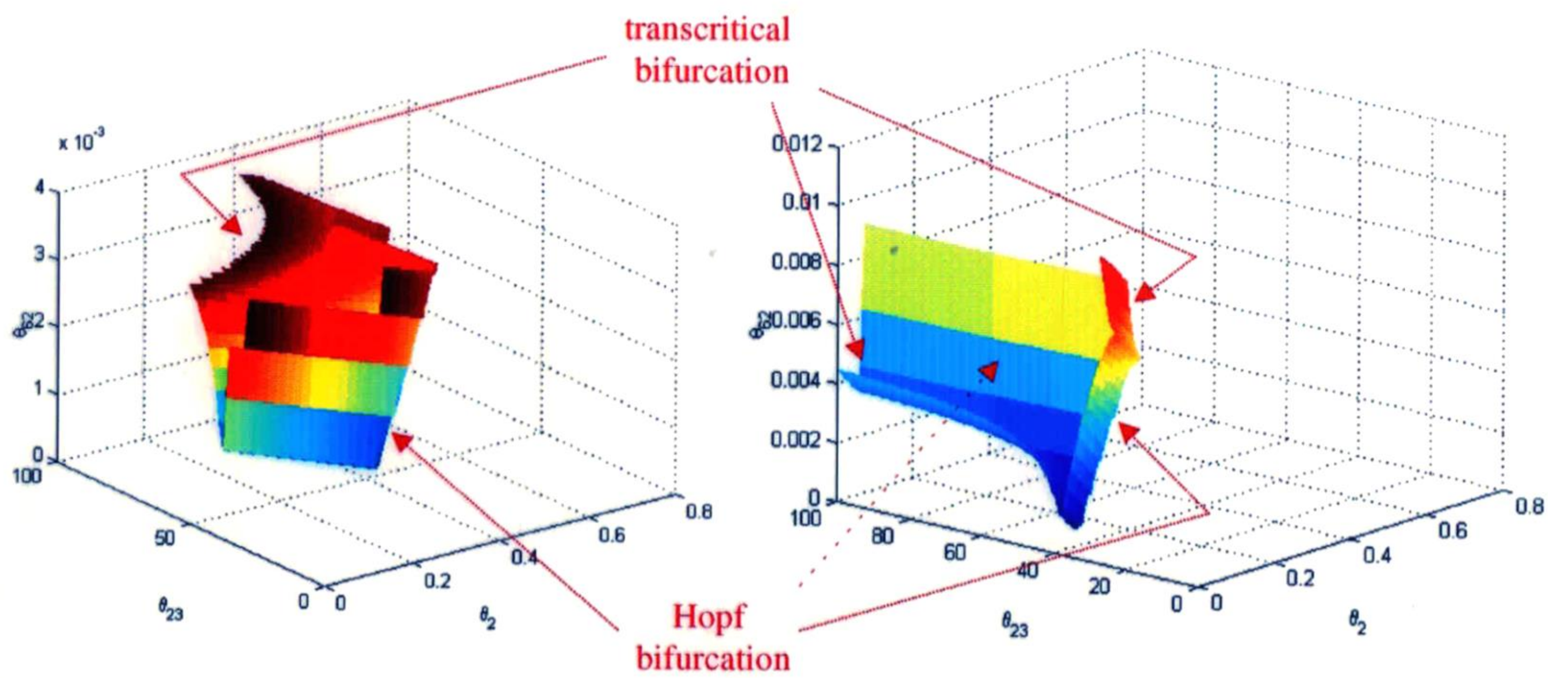
Phase Diagram for Hopf Bifurcation



Bergstrom UK Model: 3-dimensional sections of stable region

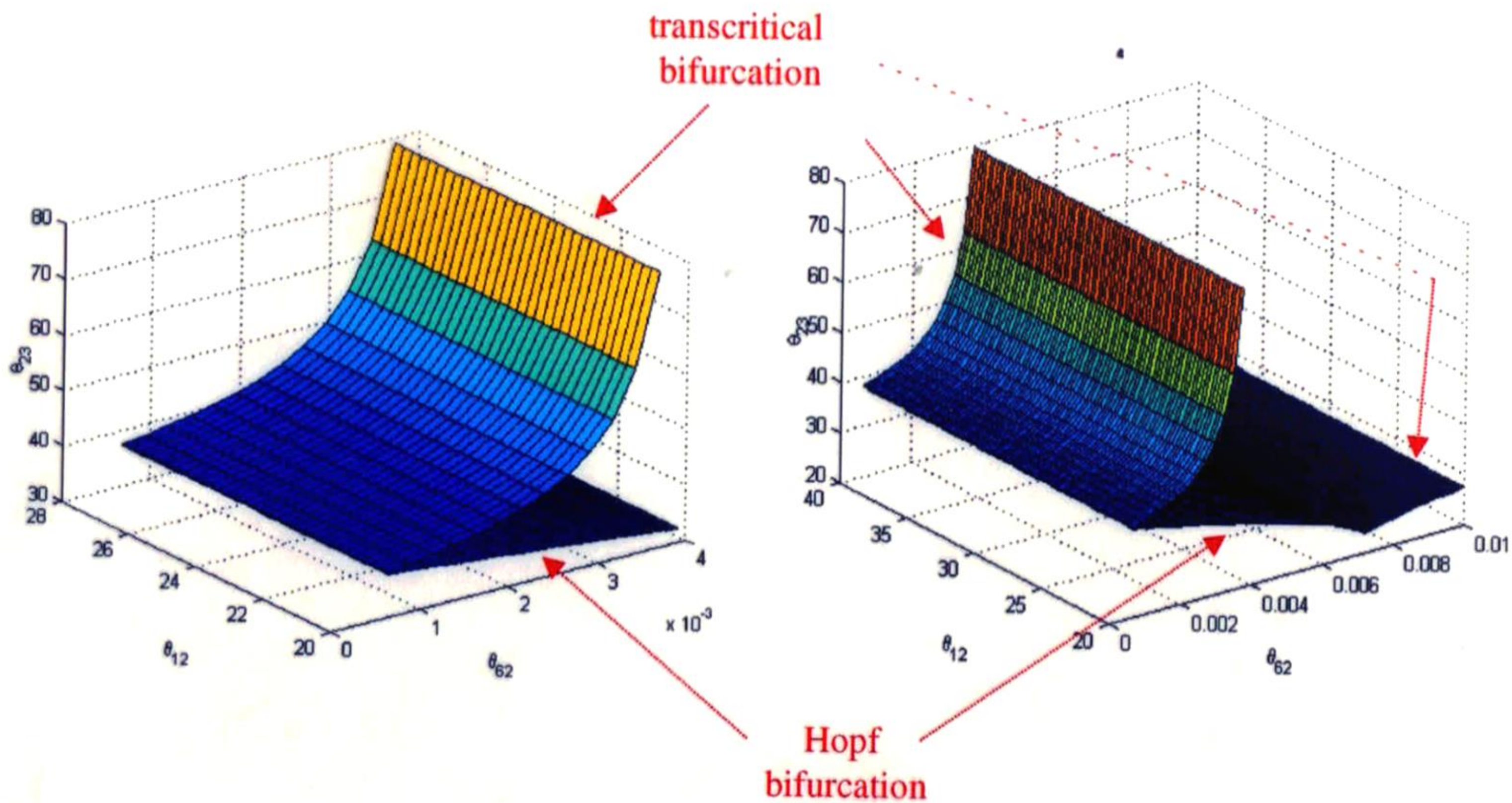
- Θ_1 is the confidence region.
- Θ is the theoretically feasible region.

$\vartheta_2, \vartheta_{23}, \vartheta_{62}$



(a) Bifurcation within Θ_1 (b) Bifurcation within Θ

$\vartheta_{12}, \vartheta_{23}, \vartheta_{62}$



(a) Bifurcation within Θ_1

(b) Bifurcation within Θ

Leeper and Sims Euler Equations Model of the US Economy

- Deep parameters solve Lucas critique.
- Model first appeared in:

  Eric Leeper and Christopher Sims, "Toward a Modern Macro Model Usable for Policy Analysis," *NBER Macroeconomics Annual*, 1994, pp. 81-117.

Our Findings with the Leeper and Sims Model

The Leeper and Sims model consists of differential equations with algebraic constraints. We find the existence of a ***singularity bifurcation*** boundary near the model's parameter point estimates. To our knowledge, this kind of bifurcation has not previously been observed in macroeconomics.

Singularity Bifurcation

- **On the singularity boundary**, the number of differential equations will decrease, while the number of algebraic constraints will increase. Singularity bifurcations thereby cause a **change in the order of the dynamics**.

Bifurcation of New Keynesian Models

- Research joint with Evgeniya A. Duzhak.
- Three economic agents:
  - Households
  - Firms
  - Central Banks
- Linearize around zero inflation steady state.

Linearized Model

- **Three Equations:**
  - Phillips curve relating inflation to output gap. The output gap is the gap between the actual sticky prices output and the flexible-price equilibrium output.
  - An IS (investment-savings) curve determining the output gap.
  - A monetary policy rule

Monetary Policy Rules

- Taylor rules:
  - Feed back inflation rate and output gap to set interest rate
- Inflation targeting:
  - Feed back only the inflation rate as a final target, in setting the interest rate.

Results with New Keynesian Models:
Types of Bifurcation Found with Each

Version of Rule	Taylor Rule	Taylor Rule with Interest Smoothing	Inflation Targeting
Current looking	Hopf bifurcation	Hopf bifurcation Flip bifurcation	Hopf bifurcation
Backward looking	Hopf bifurcation	Hopf bifurcation Flip bifurcation	Hopf bifurcation
Forward looking	Hopf bifurcation Flip bifurcation	No bifurcation boundaries found within theoretically feasible region.	Hopf bifurcation
Hybrid rule	Hopf bifurcation	Hopf bifurcation Flip bifurcation	Does not apply

Subsequent Published Research

June 2008

"Non-Robust Dynamic Inferences from Macroeconometric Models: Bifurcation Stratification of Confidence Regions," with Evgeniya A. Duzhak, Physica A

2010

"Empirical Assessment of Bifurcation Regions within New Keynesian Models," with Evgeniya A. Duzhak, Economic Theory, vol 45, nos 1-2

Nov. 2010

"Existence of Singularity Bifurcation in an Euler-Equations Model of the United States Economy: Grandmont was Right," with Yijun He, Economic Modelling

Sep. 2011

"Bifurcation Analysis of Zellner's Marshallian Macro Model," with Sanjibani Banerjee, Evgeniya Duzhak, and Ramu Gopalan, Journal of Economic Dynamics and Control

2013

"Hopf Bifurcation in the Clarida, Gali, and Gertler Model," with Unal Eryilmaz, Economic Modelling, vol 31

June 2013

"Bifurcation Analysis of an Endogenous Growth Model," with Taniya Ghosh, Journal of Economic Asymmetries, Elsevier

More Recent Publications

- "Stability Analysis of Uzawa-Lucas Endogenous Growth Model," with Taniya Ghosh, *Economic Theory Bulletin*, vol 2, no 1, April 2014.
- “Bifurcation of Macroeconometric Models and Robustness of Dynamical Inference,’ with Guo Chen, *Foundations and Trends in Econometrics*, vol 8, no 1-2, 2015.
- "Nonlinear and Complex Dynamics in Economics," with Apostolos Serletis and Demitre Serletis, *Macroeconomic Dynamics*, vol 19, no 8, 2015.
- "An Analytical and Numerical Search for Bifurcations in Open Economy New Keynesian Models," with Unal Eryilmaz, *Macroeconomic Dynamics*, vol 20, no 2, March 2016.
- “Structural Stability of the Generalized Taylor Rule,” with Evgeniya Duzhak, *Macroeconomic Dynamics*, June 2019.

Bifurcation to Chaos

How solve for boundary of subset of parameter space that can support chaos?

Not currently possible analytically with more than 3 parameters.

Numerical search possible, but rarely attempted.

Test Hypothesis of Chaos

Likelihood function filled with singularities as passes over null hypothesis subset.

Violates regularity conditions for all known sampling theoretic tests and for Bayesian inference.

If Chaotic Subset is Located and Tested, How Would It Be Used?

- Recall that all information about the unknown economic structure is in the fractal attractor set.
- But how produce the geometry of the fractal attract set without enormous sample size?
-

Calibration

Problems of inference have discouraged economic research on chaos using sampling theoretic or Bayesian methodology.

Calibration methodology has been used to find chaotic solutions to macroeconomic models.

Important contributions by Jess Benhabib and his coauthors. But without the ability to produce the geometry of the fractal attractor set, the economic usefulness of the inference is limited.

A Solution

While economists have been discouraged by their inability to produce the geometry of the fractal attractor set with small samples, mathematicians and physicists have moved ahead without the need for large samples.

While there are potentially many geometric patterns that could appear in fractal attractor sets, scientists in other fields have found that a few classes of chaotic dynamics can represent many cases in nature.

Once the characteristics of such a class of chaotic models are identified, **all of the properties of the chaotic dynamics become available** from the geometry of the fractal attractor.

Examples

- This source of major advances in other sciences has largely been **overlooked by economists, who have continued to be discouraged** by the potential infinite complexity of chaotic fractal attractor sets.
- For example, Li-Yorke chaos, for dynamical systems generated by interval maps; Lorentz attractor, a type of chaos from atmospherical dynamics models; Smale horse-shoe chaos, with origin in celestial mechanics; and Shilnikov chaos for a particular scenario/criterion related to Shilnikov orbits, have found **widespread use and relevancy in other fields**.

Shilnikov Chaos

We investigate Shilnikov chaos in New Keynesian macroeconomics for multiple reasons.

It can be **detected directly from the Shilnikov (1965) criterion**.

As explained by Alan Champneys (2010), "**Over the years, Shilnikov's mechanism of chaos has proven to be one of the most robust and frequently occurring mechanisms chosen by nature.**"

- Shilnikov, L. P. (1965), "A Case of the Existence of Denumerable Set of Periodic Motions," Sov. Math. Docl., 6: 163-166.
- Champneys, A. (2010), "To the Memory of L. P. Shilnikov," *The Dynamics Systems Web*.

Shilnikov Chaos

- As explained by Afraimovich et al (2014, p. 19): “Shilnikov . . . is a pivotal element of chaotic dynamics in a broad range of real-world applications. In general, the number of various models from hydrodynamics, optics, chemical kinetics, biology etc., which demonstrated the numerically or experimentally strange attractors with the characteristic . . . structure . . . was **overwhelming**. Indeed, this scenario has turned out to be **typical for a variety of systems and models of very diverse origins**.”
-
  - Afraimovich, Gonchenko, Lerman, Shilnokov, and Turaev (2014), “Scientific Heritage of L. P. Shilnikov,” Regular and Chaotic Dynamics, 19)4): 435-460.
-

Recognition of the Importance of Shilnikov Chaos is Growing in Mathematics, Physics, and Engineering

- The Shilnikov bifurcation theorem is **widely used** in physics, biology, electronic circuits, chemistry, and mechanical engineering.
- Shilnikov’s son, Andrey Shilnikov, is on the mathematics faculty at Georgia State University in Atlanta. His dissertation advisor was his famous father.

Our Results

- We produce a conventional New Keynesian macroeconomic model with Ricardian passive fiscal policy. Adjoining to the model a Taylor Rule feedback equation produces bifurcation to Shilnikov chaos.
  - Plausible settings of parameters and robust to conventional alternative structures of the New Keynesian model.

Our Results with US Data

- Because of properties of the Shilnikov chaos attractor set, interest rates will drift downwards. That drift can be viewed as an explanation for the liquidity trap phenomenon.
  - Unintentionally produced from the change in dynamics caused by the Taylor interest-rate feedback rule with New Keynesian sticky prices.

Liquidity Trap

- An alternative to the controversial “loanable funds” theory, imputing the ***declining nominal and real interest rates for nearly 40 years*** to oversaving by consumers.

- Oversaving theory in direct conflict with the usual explanation for the U.S. trade deficit with China: overspending. Prior to the pandemic, the US savings rate declined from 1975-2005, reaching low of 2.2% in July 2005.

Mathematical Chaos in the UK

"Shilnikov Chaos in the UK Economy"

by

William Barnett, Giovanni Bella, Taniya Ghosh, Paulo Mattana, and Beatrice Venturi

Model Used for the UK Economy

"Evolving Macroeconomic Dynamics in a Small Open Economy: An Estimated Markov Switching DSGE Model for the UK"

By

P. Liu and H. Mumtaz

Bank of England Working Paper No. 397, July 2010

Working Paper Version

Published Version

Journal of Money, Credit and Banking,
2011, Vol. 43, No. 7, pp. 1443-1474.

Model Design

A Markov switching open economy dynamic stochastic general equilibrium (DSGE) New Keynesian model.

UK data from 1970 Q1 to 2009 Q1

Their paper uses Bayesian methods to estimate a baseline rational expectations model with no regime switching and four further versions of the model, in which the UK economy is assumed to undergo structural changes across two regimes.

The most interesting case for us is the model in which the deep and the policy parameters are estimated independently in each regime (their model M5).

Allows us to study the onset of the Shilnikov phenomenon under different parameterizations and policy formulations.

We find that tightening of the policy rule, through the Taylor feedback coefficient, speeds up the way towards an unexpected phenomenon: chaotic dynamics.

The United Kingdom economy has undergone significant structural and economic changes in the past decades.

- The 1970s and 1980s: volatile inflation and output growth.
- Following implementation of inflation targeting in 1992:
  low inflation and output volatility.

Change in Policy Regimes

Prior to 1992: passive monetary policy.

Interest rates rose less than one-for-one with increase in inflation.

Impact of monetary policy shocks on inflation was considerable and positive.

Post 1992: inflation targeting active monetary policy.

Inflation responses to monetary policy shocks smaller and negative.

We consider various regimes, with the following two being the most revealing:

1. High inflation volatility and low monetary authority reaction to inflation.

2. High monetary authority reaction to inflation.

Model with high inflation and low reaction of central bank: stable without support of any form of Shilnikov chaos.

Model with low inflation and highly responsive central bank: large subset of parameter space supports Shilnikov chaos!

Using UK data, the width of the region of the parameter space within which monetary activism may induce the birth of a Shilnikov chaotic attractor depends upon the intensity of the Taylor feedback coefficient.

As the magnitude of interest rate feedback dynamics increases, the risk of unintended dynamics grows and becomes substantial.

No alert comes from a local analysis perspective, which keeps detecting the desirable property of uniqueness of the equilibrium.

A few years of data following the pandemic will be needed to identify the dynamical consequences of the Brexit bifurcation and any induced changes in central bank policy.

Comparison of the geometry of the fractal attractor set before and after Brexit may prove to be highly informative.

The research opportunities presented by Brexit are remarkable. But confounding with pandemic shocks may delay definitive results.

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