Comparison of Simple Sum and Divisia Monetary Aggregates in GDP Forecasting: a Support Vector Machines Approach

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Abstract

In this study we compare the forecasting ability of the simple sum and Divisia monetary aggregates with respect to U.S. gross domestic product. We use two alternative Divisia aggregates, the series produced by the Center for Financial Stability (CFS Divisia) and the ones produced by the Federal Reserve Bank of St. Louis (MSI Divisia). The empirical analysis is done within a machine learning framework employing a Support Vector Regression (SVR) model equipped with two kernels: the linear and the radial basis function kernel. Our training data span the period from 1967Q1 to 2007Q4 and the out-of-sample forecasts are performed on a one quarter ahead forecasting horizon on the period 2008Q1 to 2011Q4. Our tests show that the Divisia monetary aggregates are superior to the simple sum monetary aggregates in terms of standard forecast evaluation statistics.

Keywords: GDP forecasting; SVR; Simple Sum; Divisia.
JEL Codes: C22, E47, E50

1. Introduction

The mainstream approach to monetary policy in the last thirty years is focusing on short term interest rates and downplays the significant role of monetary aggregates. The reason for the abandonment of monetary aggregates was not theoretical but empirical: the established link between money and inflation was severed in recent decades when capital and financial markets' innovation produced new interest bearing assets that rendered monetary aggregates inaccurate and less relevant. These empirical findings put money and its growth rate in the backstage of empirical macroeconomic forecasting. The failure of money to forecast macroeconomic variables though may be explained by the "Barnett critique"[2]: the use so far, by central banks and economists in general, of the traditional simple-sum monetary aggregates. Barnett (1997) linked the deserved decline in the policy-importance of monetary aggregates to the specific restrictions of simple sum aggregation: the various monetary components are assigned an equal and intertemporally constant weight. Central banks facing inaccuracies and paradoxes using simple-sum aggregates as their key policy variable, abandoned money all together. Barnett (1978, 1980) developed and advocates the use of the theoretically

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[2] The phrase was coined by Chrystal and MacDonald (1994) and Belongia and Ireland (2012).

correct in terms of economic aggregation and index number theory Divisia[3] monetary aggregates. The simple sum aggregation method used widely by central banks, economists and investors even today, has been criticized heavily in the literature since Fisher (1922), Moroney and Wilbratte (1976), the seminal paper of Barnett (1980) and also Barnett (1997), Boughton (1981), Batten and Thornton (1985), Fisher and Fleissig, (1995), Schunk (2001), Darrat et al. (2005) and more recently, Barnett and Chauvet (2010), McCallum and Nelson (2011). As Irving Fisher (1922) puts it "…the simple arithmetic average produces one of the very worst of index numbers… The simple arithmetic [index] should not be used under any circumstances…" Barnett (1980) was the first to point out the unrealistic assumption for perfect substitution of the components of the simple sum aggregates. Taking into account the microeconomic aggregation theory, which offers an attractive alternative approach to the definition of money compared to the simple sum aggregation method, Barnett (1980) constructed with the appropriate modifications, the Divisia monetary aggregates. These aggregates were named after the Divisia index which serves to apply different weights to different assets in accordance with the degree of their contribution to the flow of monetary services in an economy.

Many recent studies use innovative new techniques in trying to forecast macroeconomic variables that build on VAR models. Pesaran et al. (2009) use a global vector autoregressive (GVAR) model for one and four quarters ahead forecasts for real output, inflation, real equity prices, exchange rates and interest rates for 33 countries. There is a number of papers that compare the forecasting performance of VAR and DSGE models: Smets & Wouters (2004), Del Negro et al. (2005), Rubaszek & Skrzypczyński (2008) and Bache, Jore, Mitchell & Vahey (2011) estimate VAR and DSGE models to forecast some key U.S. macroeconomic variables. Rubaszek & Skrzypczyński (2008) compare the forecast performance of a DSGE, the Survey of Professional Forecasters (SPF) and VAR models and come to the conclusion that the proposed DSGE model is not able to significantly outperform the SPF in forecasting output growth, inflation or interest rates in the United States. They also found that the DSGE model generates forecasts which are very close in accuracy to the SPF predictions. Binner et al. (2010) use two new in macroeconomics techniques, namely recurrent neural networks and kernel recursive least squares regression, in order to forecast the U.S. inflation with Divisia and simple sum aggregates. Many empirical studies such as Barnett, Offenbacher & Spindt (1984), Chrystal & MacDonald (1994), Belongia (1996) and more recently Barnett & Serletis (2000), Serletis (2006), Barnett, Jones & Nesmith (2008) Barnett (2009), Serletis and Rahman (2012) and Serletis, Istiak and Gogas (forth.) find strong evidence that Divisia indices outperform simple sum monetary aggregates as far as macro-variable forecasting and the link between money and macroeconomic activity is concerned. Schunk (2001) and Elger, Jones & Nilsson (2006) study the prediction accuracy on the real GDP for the U.S. using the simple sum and the Divisia monetary aggregates. They both use the VAR and RS-VAR methodologies. Schunk (2001) finds that the Divisia aggregates provide more accurate predictions of U.S. real GDP in contrast to the simple sum aggregates, especially at the broader levels of monetary aggregation. In contrast, Elger, Jones & Nilsson (2006) results do not favor the Divisia over the simple sum aggregates.

[3] François Divisia (1889-1964) was a French economist most noted for proposing and analyzing the Divisia Index and the Divisia monetary aggregates index (Divisia 1925).

In this paper, we empirically compare the forecasting ability of the simple sum and the Divisia monetary aggregates in terms of U.S. output. In doing so, we employ a machine learning approach with the use of a Support Vector Regression methodology that has not been used so far, to the best of our knowledge, in this empirical setting.

The paper is organized as follows: section 2 presents the data used and discusses the empirical methodology of the Support Vector Regression. The empirical results can be found in section 3. Section 4 is devoted in the robustness of our approach and the paper concludes in section 5.

2. Data and methodology

We use data on five levels of monetary aggregation, namely M1, M2M, M2, MZM and ALL, covering the period 1967Q1 to 2011Q4. We used these levels of monetary aggregation for the simple sum, CFS Divisia and MSI Divisia monetary aggregates. CFS Divisia stands for the Divisia monetary aggregate series that are produced and maintained by the Center for Financial Stability (CFS) program Advances in Monetary and Financial Measurement (AMFM). The simple sum, the MSI[4] Divisia and the real GDP data were obtained from the Federal Reserve Bank of St. Louis (FRED) database.

In this study we approach the forecasting problem from a Machine Learning perspective. The system was modeled using a Support Vector Regression as proposed by Vapnik (1995), which uses an ε-insensitive loss function for solving regression problems. The main idea of using the ε-insensitive loss function is to find a line that fits the current training data with a deviation less or equal to the value of the parameter ε (Figure 1). The solution is given through a minimization process. Given a training dataset $D = \{(\boldsymbol{x}_1, y_1), \dots . (\boldsymbol{x}_n, y_n)\}, \boldsymbol{x}_i \in \mathbb{R}^m, y_i \in \mathbb{R}, i = 1, \dots, n$, where y is the system dependent variable, search for vector $\boldsymbol{w}$ that satisfies $|y_i - (\boldsymbol{w}^T \boldsymbol{x}_i + b)| \leq \varepsilon, \forall i$ or minimizes the loss function:

$$\min\left(\frac{1}{2}\|\boldsymbol{w}\|^2\right) \tag{1}$$

subject to

$$|y_i - (\boldsymbol{w}^T \boldsymbol{x}_i + b)| \leq \varepsilon, \ \ i = 1, \dots, n$$

The model is evaluated using a part of the dataset that was not used in the training process often referred to as testing dataset. Quite often, however, given ε, the insensitive SVR model cannot cope with the dataset, i.e. there are remote data points outside the ε-zone. Error tolerance can be introduced to the model through the slack variables ζ_i and ζ_i^* and adjusted through a weight C: a predetermined parameter ($C > 0$) that boosts the slack variables in the loss function.

$$\min\left(\frac{1}{2}\|\boldsymbol{w}\|^2 + C\sum_{i=1}^{n}(\zeta_i + \zeta_i^*)\right) \tag{2}$$

subject to

[4] Monetary Services Indexes (MSI). Anderson, Richard G. and Barry E. Jones, "A Comprehensive Revision of the Monetary Services (Divisia) Indexes for the United States", Federal Reserve Bank of St. Louis Review, September/October 2011, pp. 325-359.

$$y_i - (\boldsymbol{w}^T \boldsymbol{x}_i + b) \leq \varepsilon + \zeta_i,$$
$$(\boldsymbol{w}^T \boldsymbol{x}_i + b) - y_i \leq \varepsilon + \zeta_i^*, \qquad i = 1, \dots, n$$
$$\zeta_i, \zeta_i^* \geq 0$$

The trade-off between the generalization ability and the accuracy of the model in the sample is set by adjusting ε and C. Parameter C plays the role of a regularization parameter: high values favor solutions with few misclassifications; low values produce low complexity solutions that are easier to generalize.

The dual form of the problem described in (2) takes the form:

$$\max\left(-\frac{1}{2} \sum_{i,j=1}^{n} (a_i - a_i^*)(a_j - a_j^*)\, \boldsymbol{x}_i^T \boldsymbol{x}_j - \varepsilon \sum_{i=1}^{n} (a_i - a_i^*) + \sum_{i=1}^{n} y_i (a_i - a_i^*) \right) \tag{3}$$

subject to

$$\sum_{i=1}^{n} (a_i - a_i^*) = 0 \qquad j = 1, \dots, n$$
$$a_j, a_j^* \in [0, C]$$

where a_j, a_j^* are the Lagrange multipliers taken from the Lagrangian of the primal objective function. The solution of (3) is

$$\boldsymbol{w} = \sum_{i=1}^{n} (a_i - a_i^*)\, \boldsymbol{x}_i \tag{4}$$

and

$$y = \sum_{i=1}^{n} (a_i - a_i^*)\, \boldsymbol{x}_i^T \boldsymbol{x} \tag{5}$$

This is often referred to as the Support Vector Machines regression expansion since $\boldsymbol{w}$ is constructed as a linear combination of the training patterns $\boldsymbol{x}_i$.

When the data points describe a nonlinear phenomenon then even the error-tolerant model cannot handle it successfully. In these cases the SVR methodology is coupled with a non-linear Kernel mapping, projecting the data points to a higher dimensional space, called feature space. A linear regression is, then, fitted in the created feature space and when found it is projected back in the data space (see Figure 3). In our experiments we used two kernels: a) the linear and b) radial basis function (RBF) defined as:

Linear Kernel $$K_1(\boldsymbol{x}, \boldsymbol{y}) = \boldsymbol{x}^T \boldsymbol{y} + c \tag{6}$$

RBF Kernel $$K_2(\boldsymbol{x}, \boldsymbol{y}) = e^{-\gamma \|x - y\|^2} \tag{7}$$

where $\gamma = \frac{1}{2\sigma^2}$ is the parameter adjusting the variance σ^2 of the Gaussian function.

The search for the optimal parameter setup in both cases was performed in a coarse-to-fine grid search using a 5-fold Cross Validation (CV) evaluation scheme. In this type of grid search, the parameters are initially evaluated in a large step grid in order to achieve a low accuracy image of the parameters performance. Then, we seek improved results using a denser grid focusing only in the parts of our research area where the model achieved top performance. We can repeat the procedure multiple times. In Figure 4, we provide a graphical representation of a three-iteration coarse-to-fine grid search. Optimum results in terms of forecasting performance are depicted with gray color. As the area becomes darker, the grid step becomes smaller and the search finer. Coarse-to-fine grid search is a lower complexity bypass of the exhaustive search in the finer level.

In a machine learning scheme, training results in overfitting when the model produced is significantly affected by possible noise in the sample in hand instead of the true underlying relationship that describes the phenomenon. Usually, overfitting yields a very high performance on the training step and significantly lower accuracy on the testing step. k-fold cross validation, is adopted to avoid overfitting. The dataset is cut into k chunks and the training-testing steps are repeated k times. In each turn a different chunk is used as the test dataset, while the rest k-1 chunks are grouped together to form the training dataset. The model is evaluated by averaging the performance of the model on every fold.

The training dataset spans from the first quarter of 1967 to the last quarter of 2007, while the testing dataset[5] includes the period 2008Q1-2011Q4. We used four lags of the real GDP and each monetary aggregate as independent variables in our tests in the effort to forecast real GDP one quarter ahead.

$$ln(y_t) = \Phi\left[\left(ln(y_{t-i}), ln(m_{t-i})\right)_{i=1}^{4}\right]$$

where y is the real GDP and m represents the corresponding monetary aggregate used in forecasting. According to the above, we select an optimum in terms of in-sample accuracy set of parameters for each one of the fifteen monetary aggregates used and for each of the two kernels.

The next step is to evaluate the optimized forecasting models obtained above in out-of-sample forecasting. As described before, we reserved the period 2008Q1-2011Q4 data for this purpose. These out-of-sample forecasts are compared to each other and evaluated by the root mean square error (RMSE) and mean absolute percentage error (MAPE) metrics. Thus, we produced these forecasting error metrics for the optimized (in terms of in-sample forecasting accuracy) parameter values for all three monetary aggregates (simple-sum, CFS Divisia and MSI Divisia), for all five levels of monetary aggregation (M1, M2M, M2, MZM and ALL) and for both kernels (linear and RBF).

3. Empirical Results

First, we perform a grid search for the optimum parameter values as it is described in the previous section. According to this procedure, the parameter values used and the total number of models tested are presented in Table 1. In coarse search we tested 45,000 combinations of parameter values for the linear kernel and 562,500 in the case of the RBF kernel. In the fine search that followed we tested 275,799

[5] The out-of-sample testing data were not used in the training and testing steps. Many scientists accept the in-sample testing as a second step in the training process, and the out-of-sample testing as the real model evaluation.

parameter values in the linear and 573,094 combinations for the RBF kernel. The total number of parameter setups tested was 1,456,393. The results for the linear and the RBF kernel are presented in Table 2 and 3 respectively, below. An asterisk denotes the minimum forecast error observed for each of the three monetary aggregates, simple-sum, CFS Divisia and MSI Divisia. In Tables 2 and 3 we report both the RMSE and MAPE criteria for out-of-sample forecasting accuracy for the linear and RBF kernels respectively. In Table 2, the qualitative results are exactly the same for both criteria and thus we continue the analysis with RMSE as it has a direct economic interpretation with our data: it represents the average percentage error of real GDP forecasting since in our sample all variables are expressed in natural logs. The best forecasts are obtained with M1 for the CFS and MSI Divisia aggregates with RMSE values 0.0084531 and 0.0085897 respectively and for ALL for the simple-sum aggregates with a RMSE of 0.0085843. Overall, the best accuracy in forecasting next quarter's real GDP is obtained by the CFS M1 Divisia series. With the RBF kernel in Table 3, the best forecasts are with the CFS Divisia MZM, the MSI Divisia M1 and the simple-sum M2M aggregates with RMSE values of 0.009502, 0.009275 and 0.010803 respectively. Taking all these results into account together, we can see that we obtain the best out-of-sample forecasting accuracy of real GDP when the CFS Divisia monetary aggregate series is used in M1 level of aggregation. The RMSE of the best fit CFS Divisia M1 monetary aggregate implies a forecasting average percentage error of 0.84% for the sixteen out-of-sample quarters tested. In all cases the Divisia monetary aggregates appear to be superior to the simple-sum ones.

These results provide empirical evidence using the SVR methodology in support of the *Barnett Critique* and the relevant theoretical literature that stresses the superiority of the Divisia aggregates over the widely used simple-sum ones.

4. Robustness

In this section we perform a series of robustness tests to our empirical findings of Section 3.

First, as our full data sample includes the period of the financial crisis, i.e. 2008Q1 to 2009Q2, we test the robustness of our results when the crisis period is excluded from our data sample. By re-running the best selected SVR models[6] for the CFS Divisia, the MSI Divisia and the simple sum monetary aggregates excluding the period of the crisis (2008Q1-2009Q2) we conclude that a) for the linear kernel the best forecasts are obtained using the CFS Divisia M1 aggregate with an RMSE value of 0.0147 and b) for the RBF kernel the smallest RMSE value is 0.0141 for the CFS Divisia MZM.

Second, we use a truncated sample spanning the period 1967Q1 to 2007Q4 so that we limit our complete sample just before the crisis. We employ only the best selected (in the previous section) levels of aggregation for each one of the CFS, MSI Divisia and simple sum aggregates, namely the CFS M1, MSI M2M and SS ALL. We find that for both the linear and the RBF kernel the best GDP forecasts are obtained using the MSI Divisia M1 aggregate with RMSE values 0.003256 and 0.02763 respectively.

[6] Using the set of parameters selected in the best models obtained in Section 3.

Finally, in the effort to control for other variables that may affect our systems, we also re-run the best selected SVR models[7] for the CFS Divisia, the MSI Divisia and the Simple Sum monetary aggregates but we now include the 3-month treasury bill (and its four lags) as an explanatory variable. The linear kernel provides the most accurate GDP forecasts using the MSI M1 aggregate with a 0.01163 RMSE value, while for the RBF kernel the best forecasts are obtained using the CFS M1 aggregate with an RMSE value of 0.01150. The robustness tests results are summarized in Table 4.

We can see that the best results with the linear and the RBF kernel are obtained for all the best levels of aggregation when we used the 1967-2007 dataset. Taking into consideration all the robustness tests conducted we conclude that the best forecasts were made using the M1 level of the MSI monetary aggregate with a 0.003256 RMSE value.

5. Conclusion

The aim of this study was to compare the out-of-sample forecasting accuracy of the widely used simple sum monetary aggregates against two types of the theoretically correct Divisia monetary aggregates (CFS Divisia and MSI Divisia) in terms of U.S. output. We adopted a machine learning approach employing the Support Vector Regression (SVR) technique in order to construct a model with strong generalization ability and accuracy in the training data. In doing so, we also used two alternative kernels, the linear and the RBF. According to the forecasting evaluation criteria used, the empirical evidence supports the *Barnett Critique* for the superiority of the Divisia monetary aggregates with respect to the simple sum ones. Using the linear kernel the CFS Divisia provide the most accurate out-of-sample forecasts while with the RBF kernel best accuracy was achieved with the MSI Divisia.

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[7] Using the set of parameters selected in the best models obtained in Section 3.

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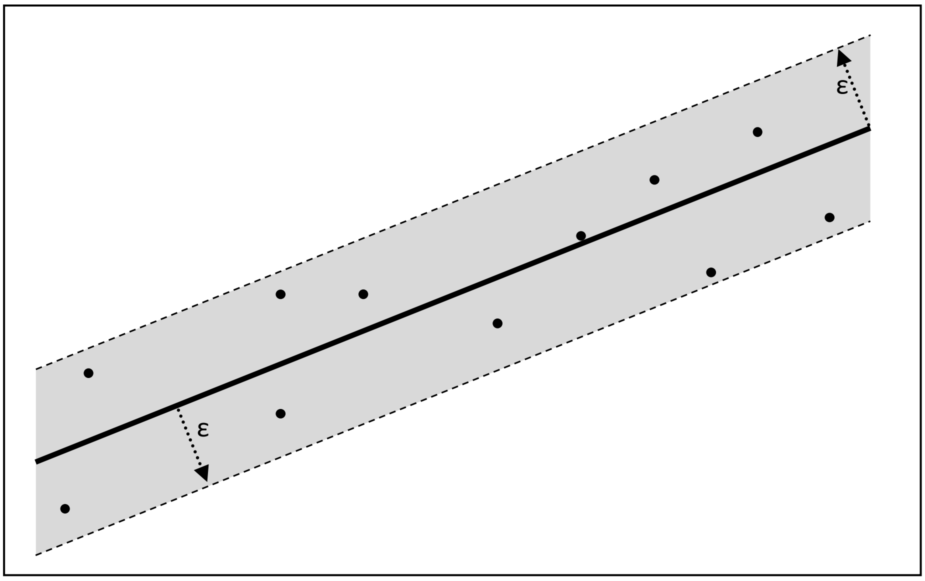

Figure 1: The ε-insensitive SVR model. All the data points should lie inside the $\pm\boldsymbol{\varepsilon}$ gray zone.

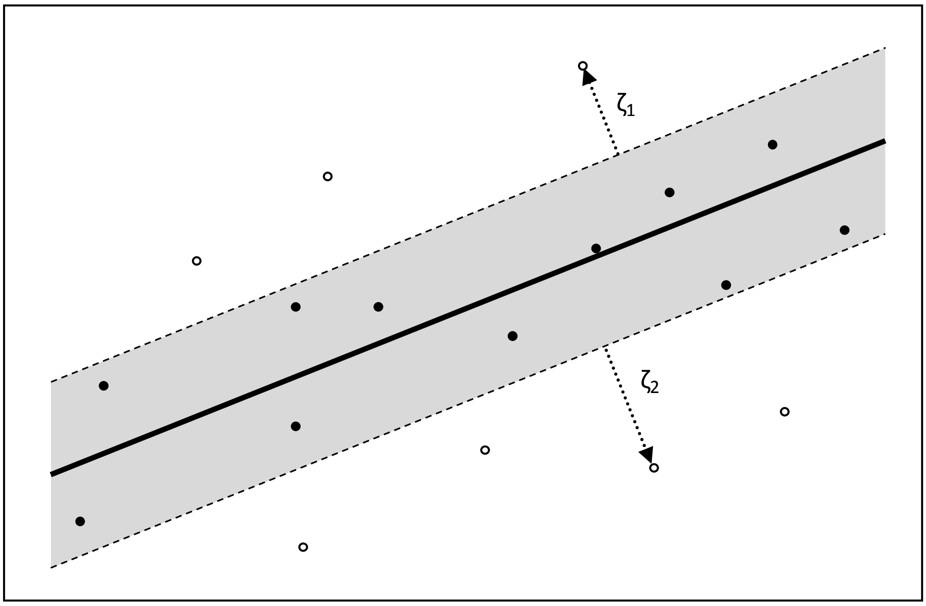

Figure 2: The error tolerance paradigm. The errors $\boldsymbol{\zeta_i}$ are multiplied by *C* in the loss function.

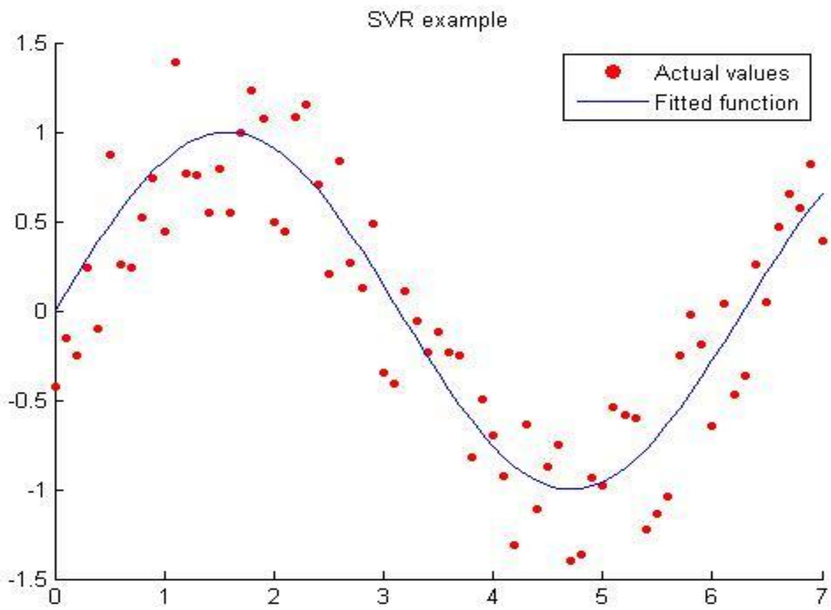

Figure 3: A nonlinear SVR case in a 2 dimensional data space

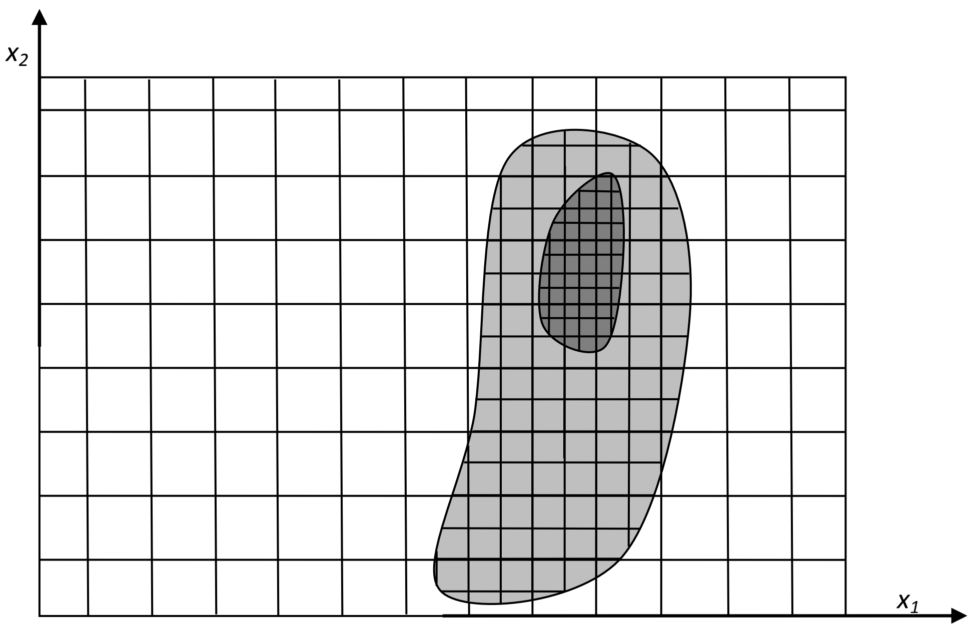

Figure 4: The coarse-to-fine grid search. From the coarser search, we advance to denser ones as the regions become darker (when darker represents better results).

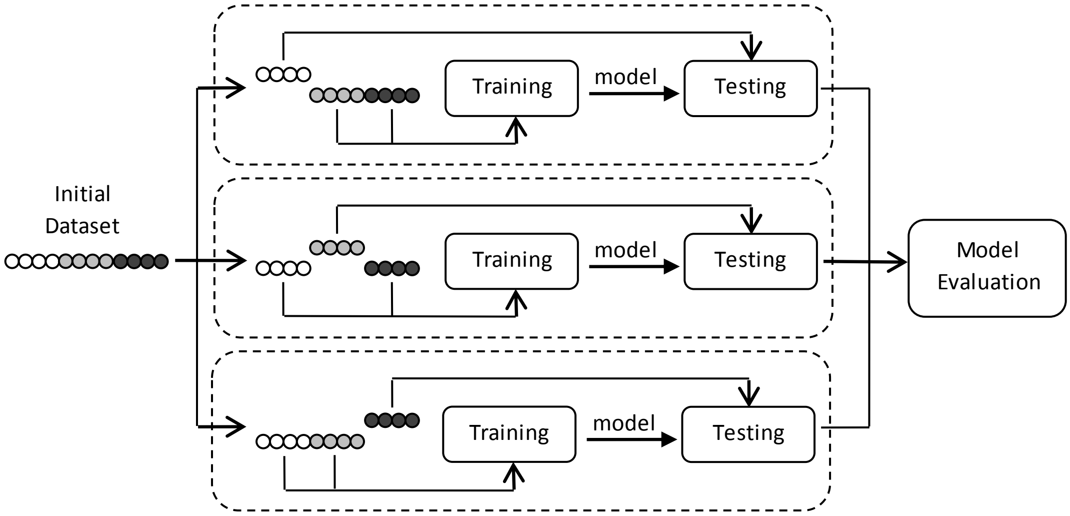

Figure 5: Overview of a 3-fold Cross Validation Evaluation System. It shows that each subset is used as a testing sample while the others are used for training the model.

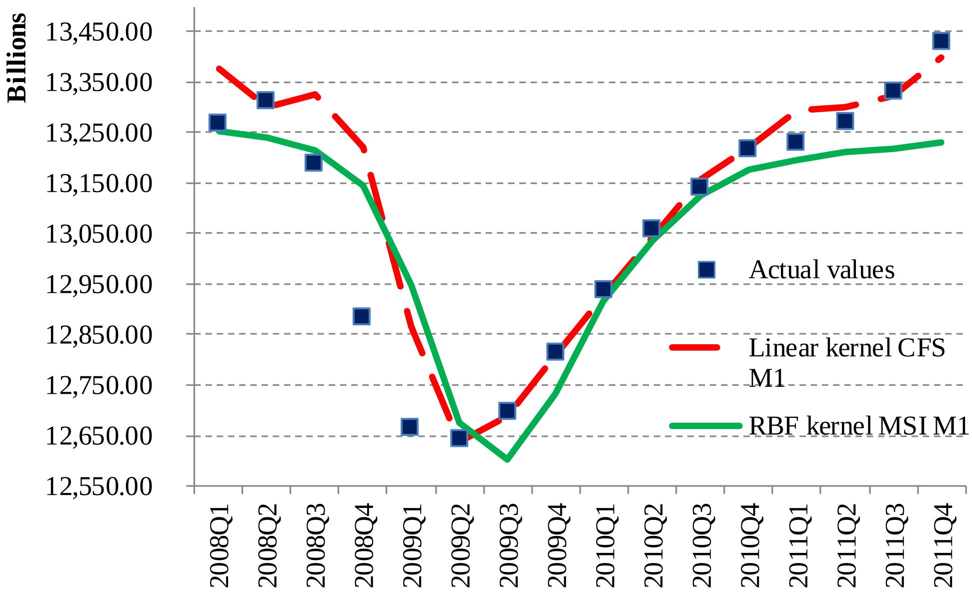

Figure 6: A nonlinear SVR case in the data space. Actual and forecasted values based on the best model and best monetary aggregate for each kernel.

Table 1. Optimum Kernel Parameters Grid Search. Total combinations tested for linear and RBF kernel in coarse and fine search.

Panel A. Coarse Search				
Parameter	Min	Max	Step	Total
Linear Kernel				
c	1	300	2	150
ε	0.0001	0.0040	0.0002	20
	Total combinations per monetary aggregate			3,000
	(A) Total combinations for 15 aggregates			**45,000**
RBF Kernel				
c	2	300	2	150
g	0.1	5.0	0.2	25
ε	0.0001	0.0020	0.0002	10
	Total combinations per monetary aggregate			37,500
	(B) Total combinations for 15 aggregates			**562,500**
Panel B. Fine Search				
Linear Kernel				275,799
RBF Kernel				573,094
	(C) Total combinations in fine search			**848,893**
	(A) + (B) + (C) Total combinations tested			**1,456,393**

Table 2. Optimal parameters (c and e) and error statistics obtained for each level of monetary aggregates for the linear kernel

Aggregate	Best c	Best e	Test RMSE	Test MAPE
CFS Divisia				
CFS M1	14.61	0.00010	0.0084531*	0.0005040 *
CFS M2M	209.70	0.00022	0.0097666	0.0006609
CFS M2	11.80	0.00036	0.0095490	0.0006474
CFS MZM	69.60	0.00038	0.0095541	0.0006612
CFS ALL	10.48	0.00001	0.0092773	0.0006312
MSI Divisia				
MSI M1	14.02	0.00170	0.0085897 *	0.0005470 *
MSI M2M	147.00	0.00030	0.0101736	0.0007097
MSI M2	101.80	0.00009	0.0093290	0.0006380
MSI MZM	250.00	0.00050	0.0092073	0.0006231
MSI ALL	227.00	0.00030	0.0088829	0.0006266
Simple Sum				
SS M1	17.28	0.00130	0.0086350	0.0005867
SS M2M	4.89	0.00410	0.0094765	0.0006339
SS M2	6.94	0.00150	0.0094031	0.0006220
SS MZM	5.42	0.00270	0.0089999	0.0006044
SS ALL	30.50	0.00230	0.0085843 *	0.0005804 *

Table 3. Optimal parameters (c, g and e) and error statistics obtained for each level of monetary aggregates for the RBF kernel

Aggregate	Best c	Best g	Best e	Test RMSE	Test MAPE
CFS Divisia					
CFS M1	129.6	0.02	0.00001	0.010183	0.000666
CFS M2M	304.0	0.79	0.00001	0.036435	0.003035
CFS M2	38.0	0.09	0.00001	0.012004	0.000981
CFS MZM	49.9	0.03	0.00001	0.009502 *	0.000601 *
CFS ALL	34.0	0.10	0.00001	0.011990	0.000944
MSI Divisia					
MSI M1	304.7	0.12	0.00001	0.009275 *	0.000695
MSI M2M	44.0	0.09	0.00001	0.010466	0.000674 *
MSI M2	305.0	0.34	0.00001	0.013329	0.001046
MSI MZM	45.0	0.03	0.00001	0.010026	0.000703
MSI ALL	302.0	5.20	0.00230	0.076714	0.006279
Simple Sum					
SS M1	302.0	5.20	0.00230	0.040484	0.003715
SS M2M	302.3	0.20	0.00001	0.010803 *	0.000850 *
SS M2	297.1	1.78	0.00001	0.054958	0.004883
SS MZM	303.0	0.24	0.00001	0.016787	0.001639
SS ALL	303.0	2.21	0.00001	0.043250	0.003781

Table 4. RMSE results for 3 robustness tests using the Linear and the RBF kernel out of sample

	Aggregate	LINEAR	Aggregate	RBF
		CFS Divisia		
Crisis dates excluded	CFS M1	0.014700	CFS MZM	0.0140508
TB3 included	CFS M1	0.014600	CFS MZM	0.0115060*
1967-2007 dataset	CFS M1	0.003549*	CFS MZM	0.0494548
		MSI Divisia		
Crisis dates excluded	MSI M1	0.014902	MSI M1	0.0284028
TB3 included	MSI M1	0.011632	MSI M1	0.0860831
1967-2007 dataset	MSI M1	0.003256*	MSI M1	0.0276277*
		Simple Sum		
Crisis dates excluded	SS ALL	0.015590	SS M2M	0.0750073
TB3 included	SS ALL	0.014406	SS M2M	0.1348513
1967-2007 dataset	SS ALL	0.003372*	SS M2M	0.0336841*