The Case for Divisia Money Targeting*

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Forthcoming in: *Macroeconomic Dynamics*

September 6, 2011

*We would like to thank William A. Barnett for his comments and suggestions. We are also grateful to Barry Jones for providing us with the simple-sum and Divisia data used in this paper.

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Abstract

In this paper we build on recent work by Serletis and Shahmoradi (2006) and Serletis and Rahman (2009) and investigate the relationship between money growth uncertainty and the level of economic activity in the United States. We pay explicit attention to the Divisia monetary aggregates, originated by Barnett (1980). In doing so, we use the new vintage of the data [called MSI (monetary services indices) by the St. Louis Fed], documented in detail by Anderson and Jones (2011), together with the simple sum monetary aggregates, over the period from 1967:1 to 2011:3. In the context of a bivariate VARMA, GARCH-in-Mean, asymmetric BEKK model, we show that increased Divisia money growth volatility (irrespective of the level of aggregation and the method of calculation) is associated with a lower average growth rate of real economic activity. However, there are no effects of simple-sum money growth volatility on real economic activity, except with the Sum M1 and perhaps Sum M2M aggregates. We conclude that monetary policies that focus on the Divisia monetary aggregates and target their growth rates will contribute to higher overall economic growth

JEL classification: C32, E52, E44.

Keywords: Multivariate GARCH-in-Mean; Monetary aggregation; Divisia money.

1 Introduction

The mainstream approach to monetary policy is based on the new Keynesian model and is expressed in terms of short-term nominal interest rates, such as the federal funds rate in the United States. However, in the aftermath of the subprime financial crisis and the Great Recession, the federal funds rate has hardly moved at all, while Federal Reserve monetary policy has been the most volatile and extreme in its entire history. This has discredited the federal funds rate as an indicator of policy and led the Fed to look elsewhere. In particular, the Fed and many central banks throughout the world have departed from the traditional interest-rate targeting approach to monetary policy and are now focusing on their balance sheet instead, using quantitative measures of monetary policy, such as credit easing and quantitative easing.

In this regard, recent empirical research regarding the relationship between the money supply and real economic activity has focused on monetary aggregation issues, motivated by Barnett's (1980) seminal paper, and the role of uncertainty or variability of money growth — see, for example, Serletis and Shahmoradi (2006) and Serletis and Rahman (2009). In particular, Serletis and Shahmoradi (2006) test Friedman's (1983, 1984) hypothesis that the variability of money growth helps predict velocity, using recent advances in the macroeconometrics literature. They find that the volatility of unanticipated money growth has a more systematic causal relation to the velocity of money than other measures of volatility and provide evidence in support of Friedman's hypothesis. More recently, Serletis and Rahman (2009) extend the work in Serletis and Shahmoradi (2006) by investigating the effects of money growth uncertainty on real economic activity, in the context of a structural vector autoregression (VAR) that is modified to accommodate multivariate GARCH-in-Mean errors. They find evidence that money growth uncertainty has significant negative effects on output growth.

In this paper, we extend the work in Serletis and Shahmoradi (2006) and Serletis and Rahman (2009) and use a more general bivariate VARMA (vector autoregressive moving average), GARCH-in-Mean, asymmetric BEKK model, as detailed in Engle and Kroner (1995), Grier *et al.* (2004), and Shields *et al.* (2005). The VARMA framework allows us to capture features of the data generating process in a more parsimonious way, without adding a large number of parameters (or lagged variables). We investigate the relationship between money growth volatility and output growth in the United States, using monthly data for simple-sum and Divisia monetary aggregates at five levels of aggregation — M1, M2, M2M, MZM, and All. Our sample period, extending from 1967:1 to 2011:3, includes the increased volatility in money supply in the aftermath of the subprime financial crisis and the global recession and also uses the new vintage of the Divisia data (called MSI by the St. Louis Fed), documented in detail in Anderson and Jones (2011).

We show that the conditional variance-covariance process underlying output and money growth exhibits significant non-diagonality and asymmetry and present evidence that Divisia money growth volatility has significant negative effects on economic activity. In particular, we show that increased uncertainty about the growth rate of Divisia money (irrespective of the level of aggregation) is associated with a lower average growth rate of real economic activity in the United States. However, there are no effects of simple-sum money growth volatility on real economic activity, except with the Sum M2 monetary aggregate. We conclude that monetary policies that focus on Divisia monetary aggregates and target their growth rates will contribute to higher overall economic growth.

The paper is organized as follows. Section 2 presents the data and Section 3 provides a brief description of the bivariate VARMA, GARCH-in-Mean, asymmetric BEKK model. Section 4 assesses the appropriateness of the econometric methodology by various information criteria and presents and discusses the empirical results. The final section concludes.

2 The Data

We use monthly United States data from the Federal Reserve Bank of St. Louis web site, over the period from 1967:1 to 2011:3, on two variables — money (M) and the industrial production index (Y). We also make comparisons among simple-sum and Divisia methods of monetary aggregation at each of five levels of aggregation — M1, M2, M2M, MZM, and the broadest available aggregation level, called All. The monetary data are the new vintage of the data, documented in detail in Anderson and Jones (2011). At the time we completed this research, Barnett's new Divisia M4 data were not yet available to the public. As Director of the Center for Financial Stability (CFS) in New York City, Barnett will soon be providing that data monthly. See www.centerforfinancialstability.org/amfm.php. In future research, we will explore the performance of the new CFS Divisia monetary aggregates.

A battery of unit root and stationarity tests are conducted in the logarithms of Y and M as well the annualized logarithmic first differences of real output and money, the latter denoted respectively by y_t and m_t. In particular, we use the augmented Dickey-Fuller (ADF) test [see Dickey and Fuller (1981)] and, given that unit root tests have low power against relevant trend stationary alternatives, we also use Kwiatkowski *et al.* (1992) tests, known as KPSS tests, for level and trend stationarity. We find that the null hypothesis of a unit root cannot be rejected and that the null hypotheses of level and trend stationarity are rejected for the logarithms of the industrial production index and each of the ten money measures. However, the ADF tests reject that null of a unit root in the logarithmic first differences of real output and each of the monetary aggregates and the KPSS $\widehat{\eta}_{\mu}$ and $\widehat{\eta}_{\tau}$ t-statistics that test the null hypotheses of level and trend stationarity are small relative to their 5% critical values of 0.463 and 0.146 (respectively), given in Kwiatkowski *et al.* (1992). We thus conclude that the logarithmic first differences of real output and money, y_t and m_t respectively, are stationary.

To see the behavior of the different monetary aggregates, Figures 1-5 provide graphical representations of the (logged) simple-sum and Divisia money measures for the United States (using the monthly data, over the period from 1967:1 to 2011:3) at each of the five levels of monetary aggregation, M1, M2, M2M, MZM, and All. See Anderson and Jones (2011) for more details regarding the data.

3 The Model

We use a version of a VARMA, GARCH-in-Mean model in the growth rates of real output and money, y_t and m_t, respectively, as follows

$$\boldsymbol{z}_t = \mathbf{a} + \sum_{i=1}^{p} \boldsymbol{\Gamma}_i \boldsymbol{z}_{t-1} + \boldsymbol{\Psi}\sqrt{\boldsymbol{h}_t} + \sum_{l=1}^{q} \boldsymbol{\Theta}_l \boldsymbol{e}_{t-l} + \boldsymbol{e}_t \tag{1}$$

$$\boldsymbol{e}_t | \, \Omega_{t-1} \sim (\mathbf{0}, \, \boldsymbol{H}_t) \, , \qquad \boldsymbol{H}_t = \begin{bmatrix} h_{yy,t} & h_{ym,t} \\ h_{my,t} & h_{mm,t} \end{bmatrix},$$

where Ω_{t-1} denotes the available information set in period $t-1$, $\mathbf{0}$ is the null vector, and

$$\boldsymbol{z}_t = \begin{bmatrix} y_t \\ m_t \end{bmatrix}; \; \boldsymbol{e}_t = \begin{bmatrix} e_{y,t} \\ e_{m,t} \end{bmatrix}; \; \boldsymbol{h}_t = \begin{bmatrix} h_{yy,t} \\ h_{mm,t} \end{bmatrix};$$

$$\boldsymbol{\Gamma}_i = \begin{bmatrix} \gamma_{11}^{(i)} & \gamma_{12}^{(i)} \\ \gamma_{21}^{(i)} & \gamma_{22}^{(i)} \end{bmatrix}; \; \boldsymbol{\Psi} = \begin{bmatrix} \psi_{11} & \psi_{12} \\ \psi_{21} & \psi_{22} \end{bmatrix}; \; \boldsymbol{\Theta}_l = \begin{bmatrix} \theta_{11}^{(l)} & \theta_{12}^{(l)} \\ \theta_{21}^{(l)} & \theta_{22}^{(l)} \end{bmatrix}.$$

Multivariate GARCH models require that we specify volatilities of y_t and m_t, measured by conditional variances. Although several different specifications have been proposed in the literature, here we use an asymmetric version of the BEKK model, introduced by Grier *et* *al.* (2004), as follows

$$\boldsymbol{H}_t = \boldsymbol{C}'\boldsymbol{C} + \sum_{j=1}^{f} \boldsymbol{B}_j'\boldsymbol{H}_{t-j}\boldsymbol{B}_j + \sum_{k=1}^{g} \boldsymbol{A}_k'\boldsymbol{e}_{t-k}\boldsymbol{e}_{t-k}'\boldsymbol{A}_k + \boldsymbol{D}'\boldsymbol{u}_{t-1}\boldsymbol{u}_{t-1}'\boldsymbol{D}, \tag{2}$$

where $\boldsymbol{C}$, $\boldsymbol{B}_j$, $\boldsymbol{A}_k$, and $\boldsymbol{D}$ are $n \times n$ matrices (for all values of j and k), with $\boldsymbol{C}$ being a triangular matrix to ensure positive definiteness of $\boldsymbol{H}$.

This specification allows past volatilities, $\boldsymbol{H}_{t-j}$, as well as lagged values of $\boldsymbol{e}\boldsymbol{e}'$ and $\boldsymbol{u}\boldsymbol{u}'$, to show up in estimating current volatilities of output and money, where $\boldsymbol{u}_t = (u_{y,t}, u_{m,t})'$ captures potential asymmetric responses. In particular, if the central bank follows an expansionary monetary policy, we take that to be good news. We therefore capture good news about money supply by a positive money supply residual, by defining $u_{m,t} = \max\{e_{m,t}, 0\}$. We also capture bad news about output by defining $u_{y,t} = \min\{e_{y,t}, 0\}$.

There are $n + n^2(p + q + 1) + n(n+1)/2 + n^2(f + g + 1)$ parameters in (1)-(2) and in order to deal with estimation problems in the large parameter space we assume that $f = g = 1$ in equation (2), consistent with recent empirical evidence regarding the superiority of GARCH(1,1) models. See, for example, Hansen and Lunde (2005).

4 Empirical Evidence

We select the optimal values of p, q, and r in equation (1) in such a way that there is no serial correlation and ARCH effects in the standardized residuals of the model. In doing so, we choose $p = q = 2$ in equation (1) and $f = g = 1$ in equation (2). The inclusion of $\boldsymbol{h}_t$ in equation (1) allows us to investigate the effect of money growth volatility on output growth.

In Tables 1-10 we report maximum likelihood estimates of the parameters (with p-values in parentheses) and diagnostic test statistics, based on the standardized residuals,

$$\widehat{z}_{jt} = \frac{e_{j_t}}{\sqrt{\widehat{h}_{j_t}}}, \qquad \text{for } j = y, m.$$

As shown in the tables, the Ljung-Box (1979) Q-statistic for testing serial correlation cannot (in general) reject the null of no autocorrelation (at conventional significance levels) for the values and the squared values of the standardized residuals (although the margins of rejection are lower in some cases), suggesting that there is no significant evidence of conditional heteroskedasticity. In addition, the failure of the data to reject the null hypotheses of $E(z) = 0$ and $E(z^2) = 1$, implicitly indicates that (based on the standardized residuals) the bivariate VARMA, GARCH-in-Mean, asymmetric BEKK model does not bear significant misspecification error — see, for example, Kroner and Ng (1998).

The effect of the conditional volatility in the money growth rate is given by $\widehat{\psi}_{12}$ in panel A of Tables 1-10. Clearly, the null hypothesis, $H_0 : \psi_{12} = 0$, is strongly rejected at the 1% level with all the Divisia aggregates, Divisia M1, Divisia M2, Divisia M2M, Divisia MZM, and Divisia All. Also, the point estimates on the coefficient, $\widehat{\psi}_{12}$, indicate that the conditional volatility of Divisia money measures has negative effects on the growth rate of output. However, the same hypothesis, $H_0 : \psi_{12} = 0$, is rejected only with the Sum M1 and Sum M2M aggregates, with the margin of rejection being lower in the case of the latter. There are no effects of conditional volatility of money growth on real economic activity with the Sum M2, Sum MZM, and Sum All aggregates.

Turning to panel B of Tables 1-10, the diagonality restriction, $\gamma_{12}^{(i)} = \gamma_{21}^{(i)} = 0$ for $i = 1, 2,$

is rejected, meaning that the data provide strong evidence of the existence of dynamic interactions between y_t and m_t. The null hypothesis of homoskedastic disturbances requires the $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{D}$ matrices to be jointly insignificant (that is, $\alpha_{ij} = \beta_{ij} = \delta_{ij} = 0$ for all i, j) and is rejected at the 1% level or better, suggesting that there is significant conditional heteroskedasticity in the data. The null hypothesis of symmetric conditional variance-covariances, which requires all elements of the $\boldsymbol{D}$ matrix to be jointly insignificant (that is, $\delta_{ij} = 0$ for all i, j), is rejected at the 1% level or better, implying the existence of some asymmetries in the data which the model is capable of capturing. Also, the null hypothesis of a diagonal covariance process requires the off-diagonal elements of the $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{D}$ matrices to be jointly insignificant (that is, $\alpha_{12} = \alpha_{21} = \beta_{12} = \beta_{21} = \delta_{12} = \delta_{21} = 0$), and is rejected. Thus the y_t-m_t process is strongly conditionally heteroskedastic, with innovations to money growth significantly influencing the conditional variance of output growth in an asymmetric way.

We have used what Anderson and Jones (2011) refer to as the 'preferred' MSI series, which are calculated by selecting the benchmark rate of return from a set of interest rates that does not include the Baa bond rate. To investigate the robustness of our results we also use the 'alternative' MSI series, also available from FRED, which are constructed by selecting the benchmark rate of return from a set of interest rates that includes the Baa corporate bond rate. The effect of the conditional volatility in the money growth rate, $\widehat{\psi}_{12}$, is $-.244$ (.008) with Divisia-A M1, -2.810 (.000) with Divisia-A M2, $-.372$ (.000) with Divisia-A M2M, $-.655$ (.005) with Divisia-A MZM, and -2.002 (.004) with Divisia-A All, suggesting that our results are robust to how the MSI data are constructed. Although Barnett originated the upper-envelope approach to measuring the benchmark rate, as used in both of the St.

Louis Federal Reserve Bank's MSI series, Barnett more recently has adopted a more direct measure of the rate of return on capital investment, based on the Bank of Israel's approach. The new benchmark rate measure is used in the Divisia monetary aggregates to be supplied in the future by the Center for Financial Stability (CFS). Although the CFS data were not yet available at the time of completion of this research, we do not anticipate that our results would be significantly altered by the use of the new benchmark rate. The Divisia monetary quantity aggregates are highly robust to the choice of benchmark rate. The monetary user-cost aggregates are not robust to that choice, but our current research uses only the quantity aggregates.

5 Conclusion

In the context of a general bivariate VARMA, GARCH-in-Mean, asymmetric BEKK model, we investigate the effects of money growth uncertainty on real economic activity in the United States over the period from 1967:1 to 2011:3, using the new vintage of the Divisia data, documented in detail in Anderson and Jones (2011). In doing so, we provide a comparison among simple-sum and Divisia methods of monetary aggregation at each of five levels of aggregation — M1, M2, M2M, MZM, and All.

We find that our model embodies a reasonable description of the U.S. data on output and money growth and we present evidence that increased uncertainty about the growth rate of the Divisia monetary aggregates is associated with a lower average growth rate of real economic activity in the United States, consistent with earlier results by Serletis and Rahman (2009). However, there are no effects of simple-sum money growth volatility on real economic activity, except with the Sum M1 and perhaps Sum M2M monetary aggregates. Thus, the money variability/output relationship is not robust to alternative methods of aggregating monetary assets and we conclude that monetary policies that focus on the Divisia monetary aggregates and target the growth rate of these aggregates will contribute to higher overall economic growth.

This is in contrast to the mainstream approach to monetary policy which is based on the new Keynesian model and is expressed in terms of the federal funds rate in the United States. In the new Keynesian approach to monetary policy, under the assumption of sticky prices, central banks use a short-term nominal interest rate as their operating instrument, but the effects of monetary policy on economic activity stem from how long-term real interest rates respond to the short-term nominal interest rate. However, the recent collapse of stable relationships in financial markets and the decoupling of long-term interest rates from short-term interest rates has significant implications for monetary policy. Moreover, as the federal funds rate has reached the zero lower bound, the Federal Reserve has lost its usual ability to signal policy changes via changes in the federal funds rate. For these reasons, in the aftermath of the subprime financial crisis and the Great Recession, the Fed and many central banks throughout the world have departed from the traditional interest-rate targeting approach to monetary policy and are now focusing on their balance sheet instead, using quantitative measures of monetary policy, such as quantitative easing.

As the Fed has been searching for new tools to steer the United States economy in an environment with the federal funds rate at the zero lower bound and the level of excess reserves in the trillions of dollars [see, for example, Barnett (2011, Chapter 4)], no one is sure how this will unfold. However, keeping the federal funds rate unusually low for a long period introduces uncertainty about the future path of money growth and inflation. This uncertainty can be especially damaging to the economy, as it amplifies the negative response of the economy to unfavorable shocks and dampens the positive response to favorable shocks. We argue that there is a need to target the growth rate of Divisia money and our results support the conclusion by Barnett and Chauvet (2011, p. 22), as further documented by Barnett (2011), that "most of the puzzles and paradoxes that have evolved in the monetary economics literature were produced by the simple-sum monetary aggregates, provided officially by most central banks, and are resolved by use of aggregation-theoretic monetary aggregates."

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Figure 1. Sum and Divisia M1 Monetary Aggregates

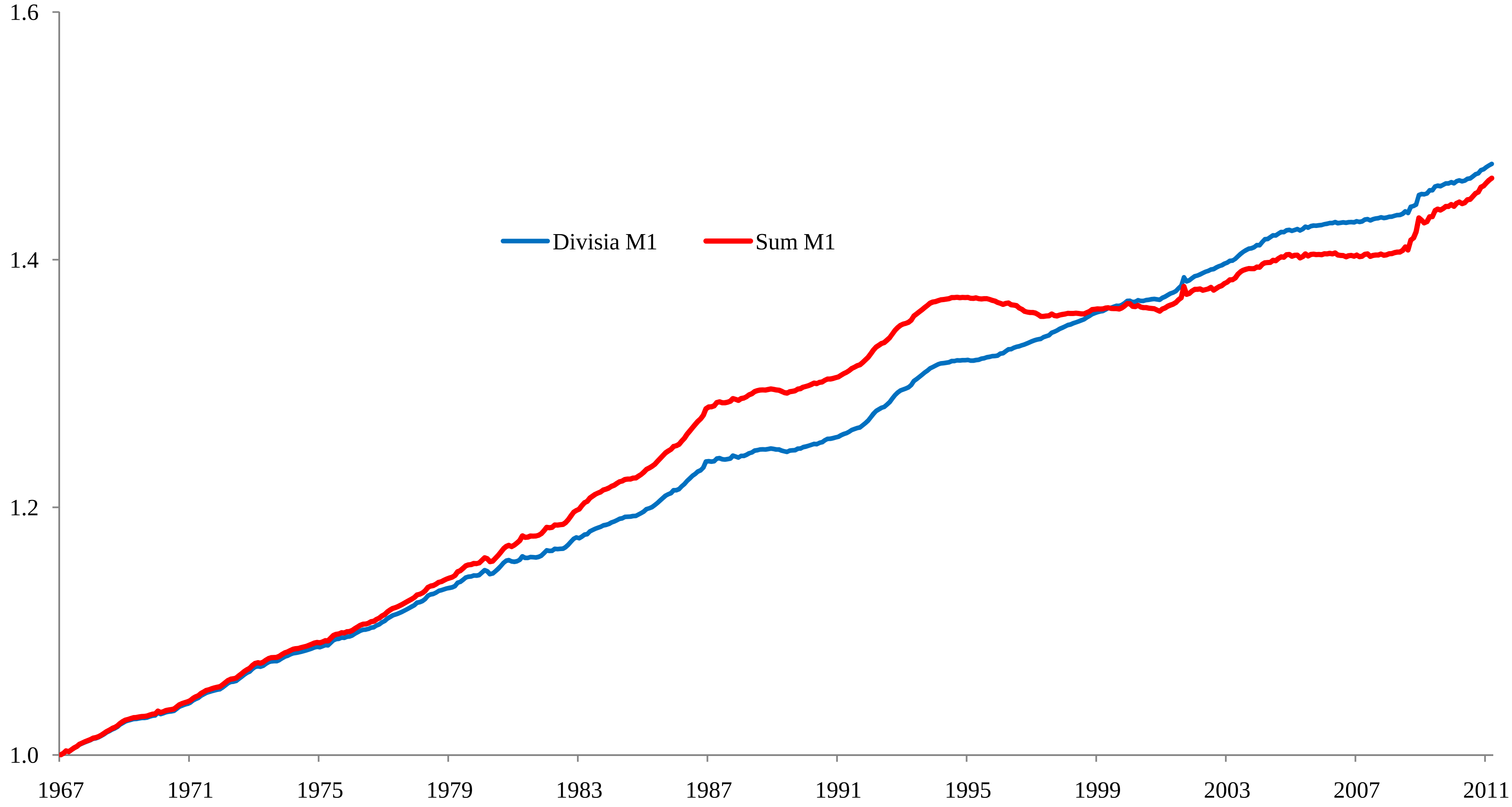

Figure 2. Sum and Divisia M2 Monetary Aggregates

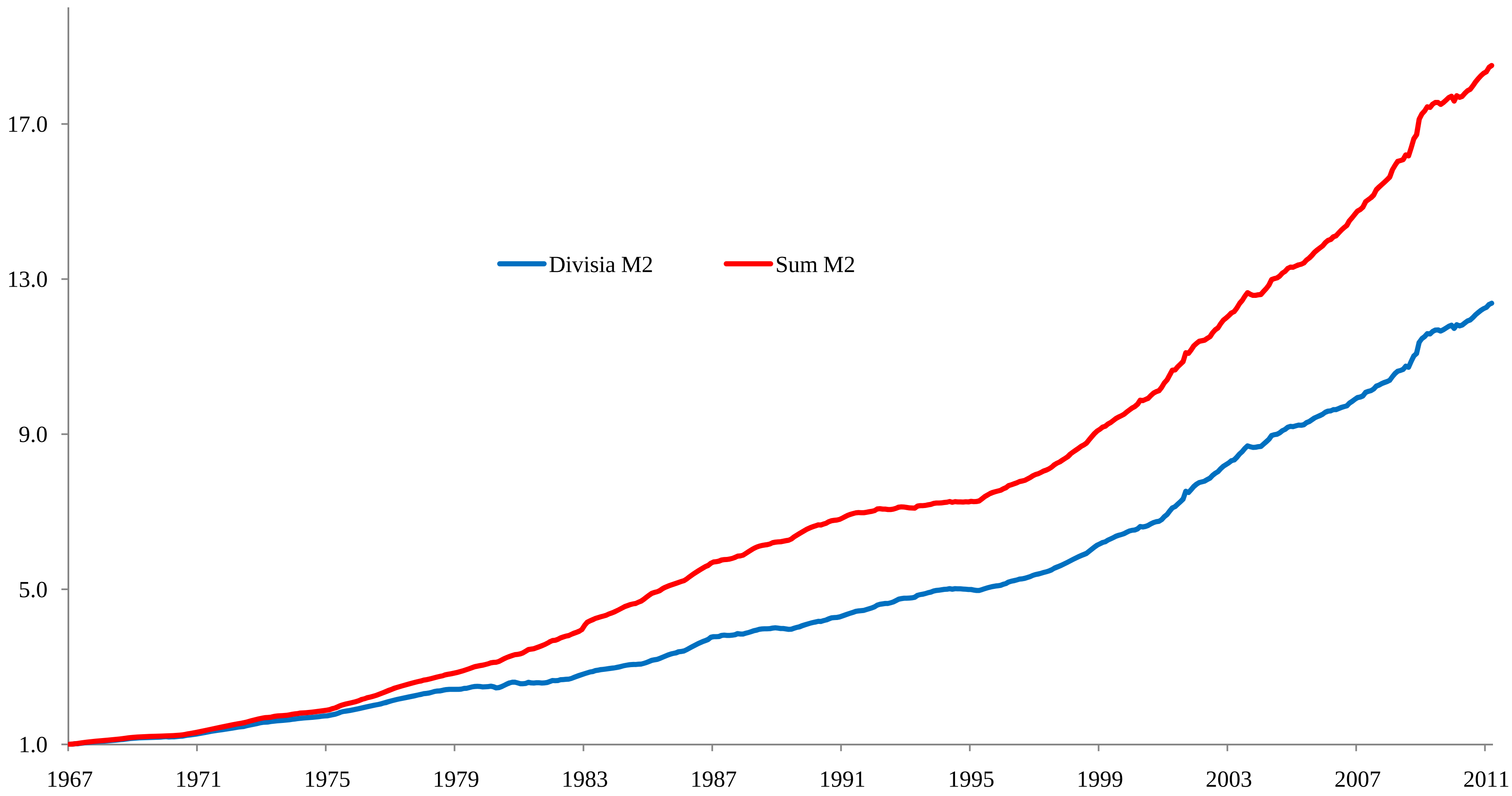

Figure 3. Sum and Divisia M2M Monetary Aggregates

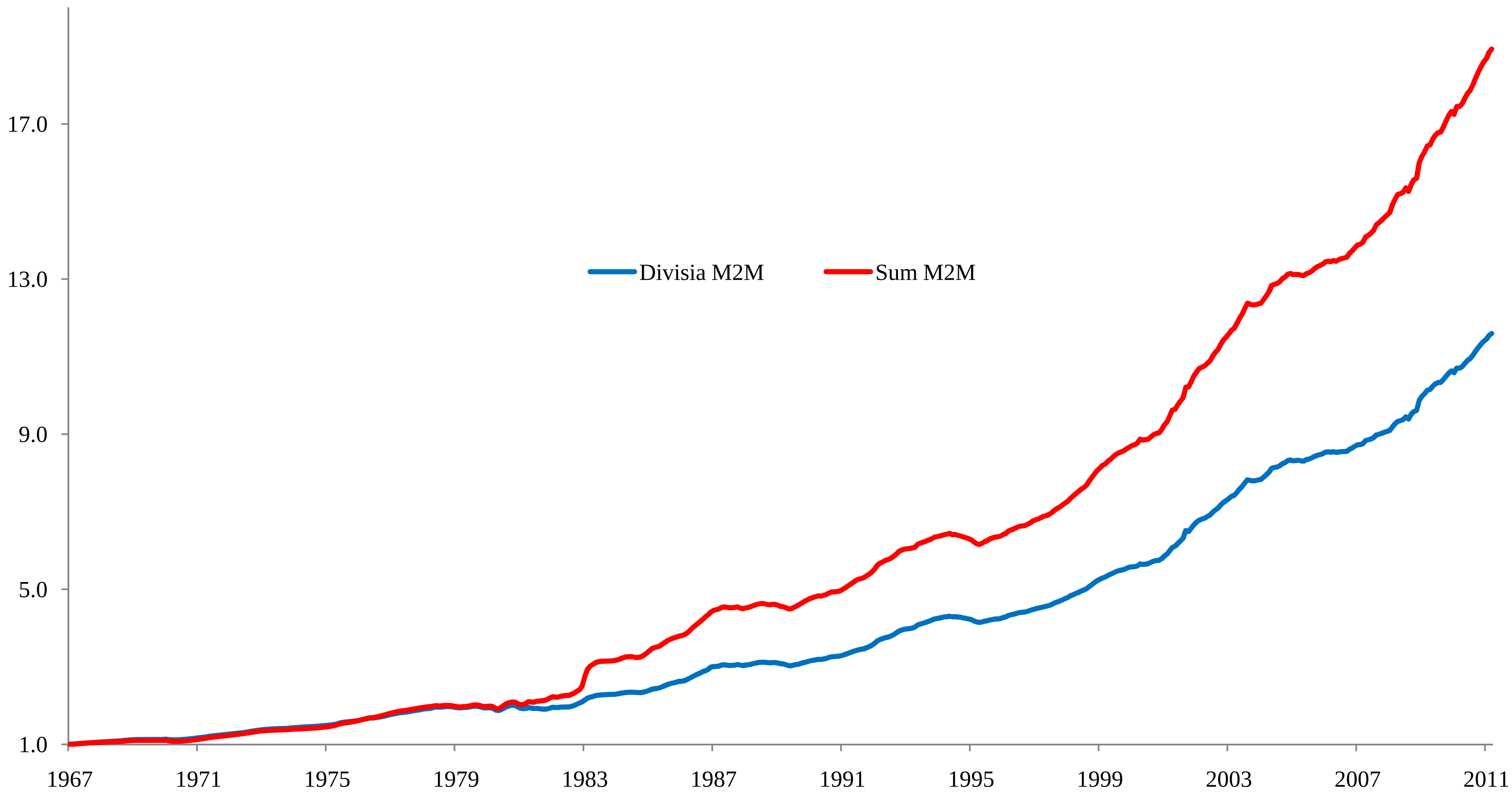

Figure 4. Sum and Divisia MZM Monetary Aggregates

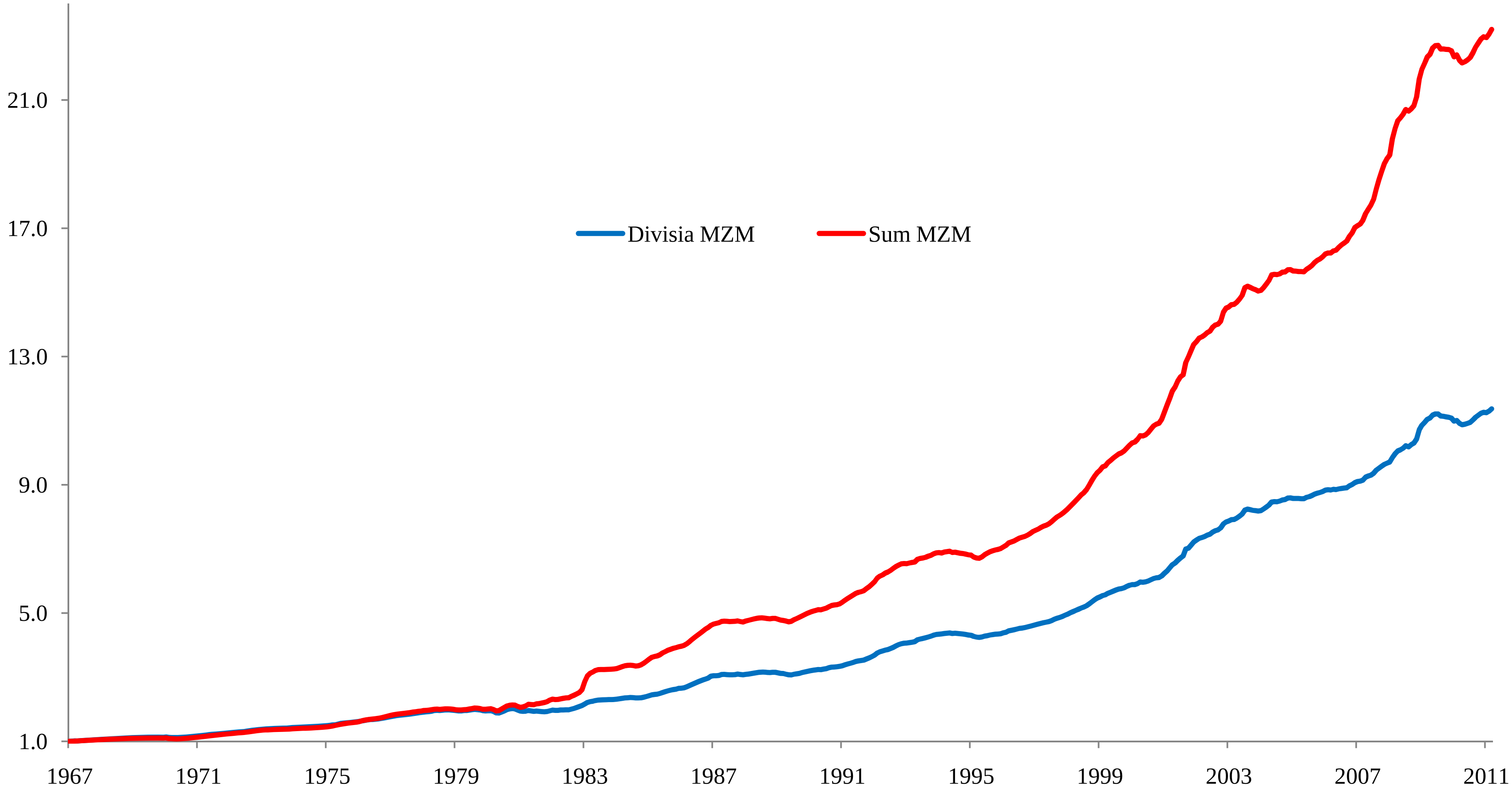

Figure 5. Sum and Divisia All Monetary Aggregates

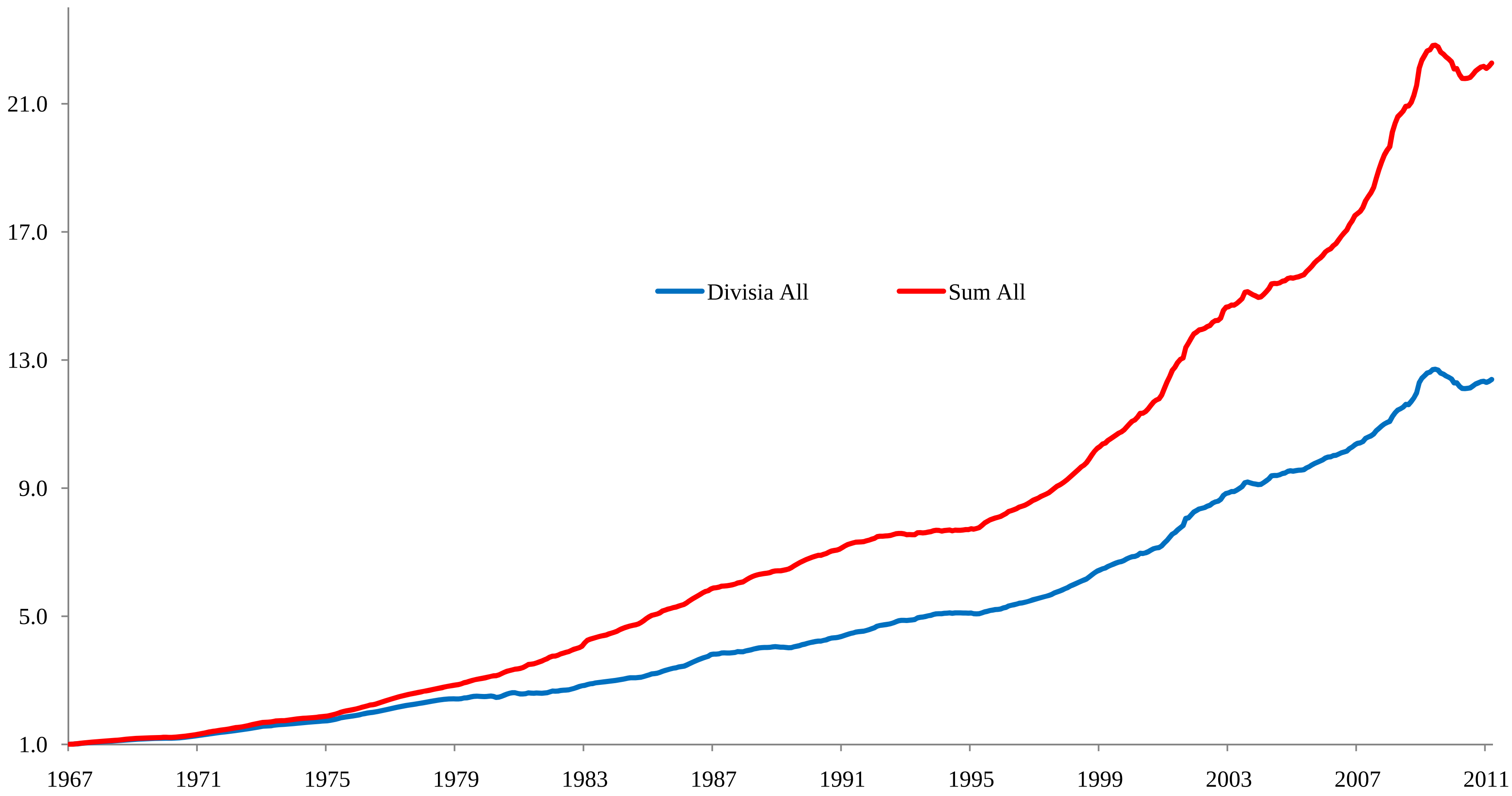

Table 1. The Bivariate Varma, Garch-in-Mean, Asymmetric BEKK Model With SUM M1 And The IP index

Equations (1) and (2) with $p = q = 2$ and $f = g = 1$

A. Conditional mean equation

$$\mathbf{a} = \begin{bmatrix} \begin{matrix} 3.559 \\ (0.000) \end{matrix} \\ \begin{matrix} 6.145 \\ (0.000) \end{matrix} \end{bmatrix}; \boldsymbol{\Gamma}_1 = \begin{bmatrix} \begin{matrix} -2.642 \\ (0.000) \end{matrix} & \begin{matrix} 1.063 \\ (0.000) \end{matrix} \\ \begin{matrix} -7.627 \\ (0.000) \end{matrix} & \begin{matrix} 3.055 \\ (0.000) \end{matrix} \end{bmatrix}; \boldsymbol{\Gamma}_2 = \begin{bmatrix} \begin{matrix} 2.590 \\ (0.000) \end{matrix} & \begin{matrix} -0.767 \\ (0.000) \end{matrix} \\ \begin{matrix} 5.804 \\ (0.000) \end{matrix} & \begin{matrix} -1.558 \\ (0.000) \end{matrix} \end{bmatrix};$$

$$\boldsymbol{\Theta}_1 = \begin{bmatrix} \begin{matrix} 2.821 \\ (0.000) \end{matrix} & \begin{matrix} -0.967 \\ (0.000) \end{matrix} \\ \begin{matrix} 7.664 \\ (0.000) \end{matrix} & \begin{matrix} -2.710 \\ (0.000) \end{matrix} \end{bmatrix}; \boldsymbol{\Theta}_2 = \begin{bmatrix} \begin{matrix} -2.043 \\ (0.000) \end{matrix} & \begin{matrix} 0.650 \\ (0.000) \end{matrix} \\ \begin{matrix} -4.588 \\ (0.000) \end{matrix} & \begin{matrix} 1.296 \\ (0.000) \end{matrix} \end{bmatrix}; \boldsymbol{\Psi}_1 = \begin{bmatrix} \begin{matrix} -0.044 \\ (0.109) \end{matrix} & \begin{matrix} -0.238 \\ (0.001) \end{matrix} \\ \begin{matrix} -0.044 \\ (0.508) \end{matrix} & \begin{matrix} -0.444 \\ (0.001) \end{matrix} \end{bmatrix}.$$

Residual diagnostics

	Mean	Variance	$Q(4)$	$Q^2(4)$	$Q(12)$	$Q^2(12)$
z_{y_t}	−0.039	1.007	12.846 (0.012)	0.763 (0.943)	22.705 (0.030)	10.252 (0.593)
z_{m_t}	0.041	1.000	9.229 (0.055)	1.662 (0.797)	22.715 (0.030)	17.273 (0.139)

B. Conditional variance-covariance structure

$$\boldsymbol{C} = \begin{bmatrix} \begin{matrix} 4.394 \\ (0.000) \end{matrix} & \begin{matrix} 1.022 \\ (0.035) \end{matrix} \\ & \begin{matrix} 1.443 \\ (0.005) \end{matrix} \end{bmatrix}; \boldsymbol{B} = \begin{bmatrix} \begin{matrix} 0.609 \\ (0.000) \end{matrix} & \begin{matrix} -0.119 \\ (0.014) \end{matrix} \\ \begin{matrix} -0.143 \\ (0.002) \end{matrix} & \begin{matrix} 0.697 \\ (0.000) \end{matrix} \end{bmatrix};$$

$$\boldsymbol{A} = \begin{bmatrix} \begin{matrix} -0.174 \\ (0.008) \end{matrix} & \begin{matrix} -0.050 \\ (0.224) \end{matrix} \\ \begin{matrix} 0.040 \\ (0.550) \end{matrix} & \begin{matrix} 0.550 \\ (0.000) \end{matrix} \end{bmatrix}; \boldsymbol{D} = \begin{bmatrix} \begin{matrix} 0.810 \\ (0.000) \end{matrix} & \begin{matrix} -0.054 \\ (0.421) \end{matrix} \\ \begin{matrix} 0.480 \\ (0.000) \end{matrix} & \begin{matrix} 0.569 \\ (0.001) \end{matrix} \end{bmatrix}.$$

Hypotheses testing

Diagonal VARMA	$H_0 : \gamma_{12}^{(i)} = \gamma_{21}^{(i)} = \theta_{12}^{(l)} = \theta_{21}^{(l)} = 0$, for $i, l = 1, 2$	0.000
No GARCH	$H_0 : \alpha_{ij} = \beta_{ij} = \delta_{ij} = 0$, for all i, j	0.000
No GARCH-M	$H_0 : \psi_{ij} = 0$, for all i, j	0.000
No asymmetry	$H_0 : \delta_{ij} = 0$, for $i, j = 1, 2$	0.000
Diagonal GARCH	$H_0 : \alpha_{12} = \alpha_{21} = \beta_{12} = \beta_{21} = \delta_{12} = \delta_{21} = 0$	0.000

Note : Sample period, monthly data: 1967:1-2011:3. Numbers in parentheses are tail areas of tests.

Table 2. The Bivariate Varma, Garch-in-Mean, Asymmetric BEKK Model With Divisia M1 And The IP index

Equations (1) and (2) with $p = q = 2$ and $f = g = 1$

A. Conditional mean equation

$$\mathbf{a} = \begin{bmatrix} \begin{array}{c} 4.895 \\ (0.000) \\ \\ 1.493 \\ (0.198) \end{array} \end{bmatrix}; \mathbf{\Gamma}_1 = \begin{bmatrix} \begin{array}{cc} 0.202 & -0.473 \\ (0.257) & (0.016) \\ & \\ -0.828 & 0.981 \\ (0.000) & (0.000) \end{array} \end{bmatrix}; \mathbf{\Gamma}_2 = \begin{bmatrix} \begin{array}{cc} 0.090 & 0.791 \\ (0.530) & (0.000) \\ & \\ 0.567 & 0.072 \\ (0.000) & (0.748) \end{array} \end{bmatrix};$$

$$\mathbf{\Theta}_1 = \begin{bmatrix} \begin{array}{cc} -0.027 & 0.620 \\ (0.895) & (0.002) \\ & \\ 0.895 & -0.665 \\ (0.000) & (0.001) \end{array} \end{bmatrix}; \mathbf{\Theta}_2 = \begin{bmatrix} \begin{array}{cc} 0.055 & -0.566 \\ (0.628) & (0.000) \\ & \\ -0.501 & -0.097 \\ (0.000) & (0.579) \end{array} \end{bmatrix}; \mathbf{\Psi}_1 = \begin{bmatrix} \begin{array}{cc} -0.247 & -0.583 \\ (0.012) & (0.000) \\ & \\ 0.015 & -0.234 \\ (0.853) & (0.051) \end{array} \end{bmatrix}.$$

Residual diagnostics

	Mean	Variance	$Q(4)$	$Q^2(4)$	$Q(12)$	$Q^2(12)$
z_{y_t}	−0.030	0.997	7.201 (0.125)	0.819 (0.935)	18.444 (0.102)	6.498 (0.818)
z_{m_t}	−0.018	0.998	15.328 (0.004)	1.295 (0.862)	45.365 (0.000)	7.729 (0.805)

B. Conditional variance-covariance structure

$$\boldsymbol{C} = \begin{bmatrix} \begin{array}{cc} 4.314 & 1.721 \\ (0.000) & (0.000) \\ & \\ & 1.684 \\ & (0.000) \end{array} \end{bmatrix}; \boldsymbol{B} = \begin{bmatrix} \begin{array}{cc} -0.686 & 0.190 \\ (0.000) & (0.000) \\ & \\ -0.113 & 0.540 \\ (0.170) & (0.000) \end{array} \end{bmatrix};$$

$$\boldsymbol{A} = \begin{bmatrix} \begin{array}{cc} -0.132 & 0.013 \\ (0.074) & (0.789) \\ & \\ -0.044 & 0.470 \\ (0.584) & (0.000) \end{array} \end{bmatrix}; \boldsymbol{D} = \begin{bmatrix} \begin{array}{cc} 0.684 & -0.006 \\ (0.000) & (0.905) \\ & \\ -0.019 & 0.707 \\ (0.888) & (0.000) \end{array} \end{bmatrix}.$$

Hypotheses testing

Diagonal VARMA	$H_0 : \gamma_{12}^{(i)} = \gamma_{21}^{(i)} = \theta_{12}^{(l)} = \theta_{21}^{(l)} = 0$, for $i, l = 1, 2$	0.000
No GARCH	$H_0 : \alpha_{ij} = \beta_{ij} = \delta_{ij} = 0$, for all i, j	0.000
No GARCH-M	$H_0 : \psi_{ij} = 0$, for all i, j	0.000
No asymmetry	$H_0 : \delta_{ij} = 0$, for $i, j = 1, 2$	0.000
Diagonal GARCH	$H_0 : \alpha_{12} = \alpha_{21} = \beta_{12} = \beta_{21} = \delta_{12} = \delta_{21} = 0$	0.000

Note : Sample period, monthly data: 1967:1-2011:3. Numbers in parentheses are tail areas of tests.

Table 3. The Bivariate Varma, Garch-in-Mean, Asymmetric BEKK Model With SUM M2 And The IP index

Equations (1) and (2) with $p = q = 2$ and $f = g = 1$

A. Conditional mean equation

$$\mathbf{a} = \begin{bmatrix} \underset{(0.117)}{-1.172} \\ \underset{(0.502)}{0.288} \end{bmatrix}; \boldsymbol{\Gamma}_1 = \begin{bmatrix} \underset{(0.173)}{-0.214} & \underset{(0.000)}{-0.489} \\ \underset{(0.000)}{1.030} & \underset{(0.000)}{1.587} \end{bmatrix}; \boldsymbol{\Gamma}_2 = \begin{bmatrix} \underset{(0.000)}{0.917} & \underset{(0.000)}{0.523} \\ \underset{(0.000)}{-0.824} & \underset{(0.000)}{-0.661} \end{bmatrix};$$

$$\boldsymbol{\Theta}_1 = \begin{bmatrix} \underset{(0.009)}{0.425} & \underset{(0.000)}{0.501} \\ \underset{(0.000)}{-1.015} & \underset{(0.000)}{-1.001} \end{bmatrix}; \boldsymbol{\Theta}_2 = \begin{bmatrix} \underset{(0.000)}{-0.697} & \underset{(0.000)}{-0.194} \\ \underset{(0.000)}{0.567} & \underset{(0.446)}{0.046} \end{bmatrix}; \boldsymbol{\Psi}_1 = \begin{bmatrix} \underset{(0.021)}{0.280} & \underset{(0.492)}{-0.129} \\ \underset{(0.244)}{-0.087} & \underset{(0.283)}{0.110} \end{bmatrix}.$$

Residual diagnostics

	Mean	Variance	$Q(4)$	$Q^2(4)$	$Q(12)$	$Q^2(12)$
z_{y_t}	−0.008	1.017	3.851 (0.426)	2.006 (0.734)	15.480 (0.215)	8.326 (0.759)
z_{m_t}	−0.023	0.994	7.398 (0.116)	4.154 (0.385)	29.444 (0.003)	9.084 (0.695)

B. Conditional variance-covariance structure

$$\boldsymbol{C} = \begin{bmatrix} \underset{(0.000)}{4.065} & \underset{(0.001)}{0.814} \\ & \underset{(0.005)}{0.799} \end{bmatrix}; \boldsymbol{B} = \begin{bmatrix} \underset{(0.000)}{-0.699} & \underset{(0.180)}{0.046} \\ \underset{(0.001)}{0.319} & \underset{(0.000)}{-0.760} \end{bmatrix};$$

$$\boldsymbol{A} = \begin{bmatrix} \underset{(0.168)}{-0.069} & \underset{(0.052)}{-0.039} \\ \underset{(0.236)}{0.094} & \underset{(0.000)}{-0.407} \end{bmatrix}; \boldsymbol{D} = \begin{bmatrix} \underset{(0.000)}{0.629} & \underset{(0.148)}{-0.048} \\ \underset{(0.342)}{-0.193} & \underset{(0.001)}{-0.614} \end{bmatrix}.$$

Hypotheses testing

Diagonal VARMA	$H_0 : \gamma_{12}^{(i)} = \gamma_{21}^{(i)} = \theta_{12}^{(l)} = \theta_{21}^{(l)} = 0$, for $i, l = 1, 2$	0.000
No GARCH	$H_0 : \alpha_{ij} = \beta_{ij} = \delta_{ij} = 0$, for all i, j	0.000
No GARCH-M	$H_0 : \psi_{ij} = 0$, for all i, j	0.001
No asymmetry	$H_0 : \delta_{ij} = 0$, for $i, j = 1, 2$	0.000
Diagonal GARCH	$H_0 : \alpha_{12} = \alpha_{21} = \beta_{12} = \beta_{21} = \delta_{12} = \delta_{21} = 0$	0.001

Note : Sample period, monthly data: 1967:1-2011:3. Numbers in parentheses are tail areas of tests.

Table 4. The Bivariate Varma, Garch-in-Mean, Asymmetric BEKK Model With Divisia M2 And The IP index

Equations (1) and (2) with $p = q = 2$ and $f = g = 1$

A. Conditional mean equation

$$\mathbf{a} = \begin{bmatrix} \begin{matrix} 7.096 \\ (0.000) \end{matrix} \\ \begin{matrix} 8.037 \\ (0.000) \end{matrix} \end{bmatrix}; \boldsymbol{\Gamma}_1 = \begin{bmatrix} \begin{matrix} 1.366 \\ (0.000) \end{matrix} & \begin{matrix} -1.783 \\ (0.000) \end{matrix} \\ \begin{matrix} 0.300 \\ (0.000) \end{matrix} & \begin{matrix} -0.069 \\ (0.000) \end{matrix} \end{bmatrix}; \boldsymbol{\Gamma}_2 = \begin{bmatrix} \begin{matrix} -0.401 \\ (0.000) \end{matrix} & \begin{matrix} 1.476 \\ (0.000) \end{matrix} \\ \begin{matrix} -0.266 \\ (0.000) \end{matrix} & \begin{matrix} 0.673 \\ (0.000) \end{matrix} \end{bmatrix};$$

$$\boldsymbol{\Theta}_1 = \begin{bmatrix} \begin{matrix} -1.172 \\ (0.000) \end{matrix} & \begin{matrix} 2.027 \\ (0.000) \end{matrix} \\ \begin{matrix} 0.362 \\ (0.000) \end{matrix} & \begin{matrix} 0.659 \\ (0.000) \end{matrix} \end{bmatrix}; \boldsymbol{\Theta}_2 = \begin{bmatrix} \begin{matrix} -0.203 \\ (0.000) \end{matrix} & \begin{matrix} -0.845 \\ (0.000) \end{matrix} \\ \begin{matrix} 0.175 \\ (0.000) \end{matrix} & \begin{matrix} -0.423 \\ (0.000) \end{matrix} \end{bmatrix}; \boldsymbol{\Psi}_1 = \begin{bmatrix} \begin{matrix} 0.543 \\ (0.000) \end{matrix} & \begin{matrix} -2.619 \\ (0.000) \end{matrix} \\ \begin{matrix} 0.326 \\ (0.000) \end{matrix} & \begin{matrix} -2.353 \\ (0.000) \end{matrix} \end{bmatrix}.$$

Residual diagnostics

	Mean	Variance	$Q(4)$	$Q^2(4)$	$Q(12)$	$Q^2(12)$
z_{y_t}	−0.034	0.996	14.200 (0.006)	2.202 (0.731)	30.337(0.002)	7.488 (0.823)
z_{m_t}	0.015	0.972	9.778 (0.044)	2.611 (0.624)	33.130 (0.000)	8.333 (0.758)

B. Conditional variance-covariance structure

$$\boldsymbol{C} = \begin{bmatrix} \begin{matrix} 4.038 \\ (0.000) \end{matrix} & \begin{matrix} 0.291 \\ (0.128) \end{matrix} \\ & \begin{matrix} 3.001 \\ (0.000) \end{matrix} \end{bmatrix}; \boldsymbol{B} = \begin{bmatrix} \begin{matrix} -0.643 \\ (0.000) \end{matrix} & \begin{matrix} -0.011 \\ (0.604) \end{matrix} \\ \begin{matrix} 0.288 \\ (0.000) \end{matrix} & \begin{matrix} -0.258 \\ (0.000) \end{matrix} \end{bmatrix};$$

$$\boldsymbol{A} = \begin{bmatrix} \begin{matrix} 0.086 \\ (0.189) \end{matrix} & \begin{matrix} -0.021 \\ (0.324) \end{matrix} \\ \begin{matrix} 0.229 \\ (0.025) \end{matrix} & \begin{matrix} 0.281 \\ (0.000) \end{matrix} \end{bmatrix}; \boldsymbol{D} = \begin{bmatrix} \begin{matrix} -0.818 \\ (0.000) \end{matrix} & \begin{matrix} 0.275 \\ (0.000) \end{matrix} \\ \begin{matrix} -0.624 \\ (0.000) \end{matrix} & \begin{matrix} 0.558 \\ (0.000) \end{matrix} \end{bmatrix}.$$

Hypotheses testing

Diagonal VARMA	$H_0 : \gamma_{12}^{(i)} = \gamma_{21}^{(i)} = \theta_{12}^{(l)} = \theta_{21}^{(l)} = 0$, for $i, l = 1, 2$	0.000
No GARCH	$H_0 : \alpha_{ij} = \beta_{ij} = \delta_{ij} = 0$, for all i, j	0.000
No GARCH-M	$H_0 : \psi_{ij} = 0$, for all i, j	0.000
No asymmetry	$H_0 : \delta_{ij} = 0$, for $i, j = 1, 2$	0.000
Diagonal GARCH	$H_0 : \alpha_{12} = \alpha_{21} = \beta_{12} = \beta_{21} = \delta_{12} = \delta_{21} = 0$	0.000

Note : Sample period, monthly data: 1967:1-2011:3. Numbers in parentheses are tail areas of tests.

TABLE 5. THE BIVARIATE VARMA, GARCH-IN-MEAN, ASYMMETRIC BEKK MODEL WITH SUM M2M AND THE IP INDEX

Equations (1) and (2) with $p = q = 2$ and $f = g = 1$

A. Conditional mean equation

$$\mathbf{a} = \begin{bmatrix} \begin{matrix} 20.587 \\ (0.000) \end{matrix} \\ \begin{matrix} 4.220 \\ (0.000) \end{matrix} \end{bmatrix}; \mathbf{\Gamma}_1 = \begin{bmatrix} \begin{matrix} 0.502 \\ (0.133) \end{matrix} & \begin{matrix} -7.171 \\ (0.000) \end{matrix} \\ \begin{matrix} -0.087 \\ (0.083) \end{matrix} & \begin{matrix} 0.343 \\ (0.007) \end{matrix} \end{bmatrix}; \mathbf{\Gamma}_2 = \begin{bmatrix} \begin{matrix} -0.505 \\ (0.193) \end{matrix} & \begin{matrix} 5.486 \\ (0.000) \end{matrix} \\ \begin{matrix} -0.064 \\ (0.003) \end{matrix} & \begin{matrix} 0.313 \\ (0.000) \end{matrix} \end{bmatrix};$$

$$\mathbf{\Theta}_1 = \begin{bmatrix} \begin{matrix} -0.376 \\ (0.305) \end{matrix} & \begin{matrix} 7.329 \\ (0.000) \end{matrix} \\ \begin{matrix} 0.077 \\ (0.139) \end{matrix} & \begin{matrix} 0.366 \\ (0.009) \end{matrix} \end{bmatrix}; \mathbf{\Theta}_2 = \begin{bmatrix} \begin{matrix} 0.562 \\ (0.154) \end{matrix} & \begin{matrix} -0.428 \\ (0.200) \end{matrix} \\ \begin{matrix} 0.054 \\ (0.011) \end{matrix} & \begin{matrix} -0.041 \\ (0.124) \end{matrix} \end{bmatrix}; \mathbf{\Psi}_1 = \begin{bmatrix} \begin{matrix} -0.572 \\ (0.062) \end{matrix} & \begin{matrix} -0.669 \\ (0.014) \end{matrix} \\ \begin{matrix} -0.154 \\ (0.000) \end{matrix} & \begin{matrix} -0.107 \\ (0.048) \end{matrix} \end{bmatrix}.$$

Residual diagnostics

	Mean	Variance	$Q(4)$	$Q^2(4)$	$Q(12)$	$Q^2(12)$
z_{y_t}	0.022	1.005	4.757 (0.313)	1.235 (0.872)	11.457 (0.490)	8.977 (0.704)
z_{m_t}	−0.005	0.988	1.638 (0.801)	9.072 (0.059)	32.136 (0.001)	12.832 (0.381)

B. Conditional variance-covariance structure

$$\boldsymbol{C} = \begin{bmatrix} \begin{matrix} 5.314 \\ (0.000) \end{matrix} & \begin{matrix} 0.111 \\ (0.676) \end{matrix} \\ & \begin{matrix} 1.417 \\ (0.101) \end{matrix} \end{bmatrix}; \boldsymbol{B} = \begin{bmatrix} \begin{matrix} 0.463 \\ (0.002) \end{matrix} & \begin{matrix} 0.132 \\ (0.102) \end{matrix} \\ \begin{matrix} -0.211 \\ (0.003) \end{matrix} & \begin{matrix} 0.776 \\ (0.000) \end{matrix} \end{bmatrix};$$

$$\boldsymbol{A} = \begin{bmatrix} \begin{matrix} -0.018 \\ (0.906) \end{matrix} & \begin{matrix} -0.070 \\ (0.351) \end{matrix} \\ \begin{matrix} 0.073 \\ (0.543) \end{matrix} & \begin{matrix} 0.455 \\ (0.000) \end{matrix} \end{bmatrix}; \boldsymbol{D} = \begin{bmatrix} \begin{matrix} -0.805 \\ (0.000) \end{matrix} & \begin{matrix} 0.241 \\ (0.003) \end{matrix} \\ \begin{matrix} -0.072 \\ (0.527) \end{matrix} & \begin{matrix} 0.541 \\ (0.010) \end{matrix} \end{bmatrix}.$$

Hypotheses testing

Diagonal VARMA	$H_0 : \gamma_{12}^{(i)} = \gamma_{21}^{(i)} = \theta_{12}^{(l)} = \theta_{21}^{(l)} = 0$, for $i, l = 1, 2$	0.000
No GARCH	$H_0 : \alpha_{ij} = \beta_{ij} = \delta_{ij} = 0$, for all i, j	0.000
No GARCH-M	$H_0 : \psi_{ij} = 0$, for all i, j	0.000
No asymmetry	$H_0 : \delta_{ij} = 0$, for $i, j = 1, 2$	0.000
Diagonal GARCH	$H_0 : \alpha_{12} = \alpha_{21} = \beta_{12} = \beta_{21} = \delta_{12} = \delta_{21} = 0$	0.006

Note : Sample period, monthly data: 1967:1-2011:3. Numbers in parentheses are tail areas of tests.

Table 6. The Bivariate Varma, Garch-in-Mean, Asymmetric BEKK Model With Divisia M2M And The IP index

Equations (1) and (2) with $p = q = 2$ and $f = g = 1$

A. Conditional mean equation

$$\mathbf{a} = \begin{bmatrix} \begin{matrix} 17.314 \\ (0.000) \end{matrix} \\ \begin{matrix} 9.066 \\ (0.000) \end{matrix} \end{bmatrix}; \mathbf{\Gamma}_1 = \begin{bmatrix} \begin{matrix} 2.818 \\ (0.000) \end{matrix} & \begin{matrix} -7.803 \\ (0.000) \end{matrix} \\ \begin{matrix} 0.699 \\ (0.000) \end{matrix} & \begin{matrix} -2.144 \\ (0.000) \end{matrix} \end{bmatrix}; \mathbf{\Gamma}_2 = \begin{bmatrix} \begin{matrix} -2.278 \\ (0.000) \end{matrix} & \begin{matrix} 6.212 \\ (0.000) \end{matrix} \\ \begin{matrix} -0.899 \\ (0.000) \end{matrix} & \begin{matrix} 2.318 \\ (0.000) \end{matrix} \end{bmatrix};$$

$$\mathbf{\Theta}_1 = \begin{bmatrix} \begin{matrix} -2.689 \\ (0.000) \end{matrix} & \begin{matrix} 7.951 \\ (0.000) \end{matrix} \\ \begin{matrix} -0.718 \\ (0.000) \end{matrix} & \begin{matrix} 2.706 \\ (0.000) \end{matrix} \end{bmatrix}; \mathbf{\Theta}_2 = \begin{bmatrix} \begin{matrix} 1.933 \\ (0.000) \end{matrix} & \begin{matrix} -2.225 \\ (0.000) \end{matrix} \\ \begin{matrix} 0.765 \\ (0.000) \end{matrix} & \begin{matrix} -0.824 \\ (0.000) \end{matrix} \end{bmatrix}; \mathbf{\Psi}_1 = \begin{bmatrix} \begin{matrix} -0.123 \\ (0.000) \end{matrix} & \begin{matrix} -1.676 \\ (0.000) \end{matrix} \\ \begin{matrix} -0.104 \\ (0.000) \end{matrix} & \begin{matrix} -0.841 \\ (0.000) \end{matrix} \end{bmatrix}.$$

Residual diagnostics

	Mean	Variance	$Q(4)$	$Q^2(4)$	$Q(12)$	$Q^2(12)$
z_{y_t}	−0.031	0.992	7.383 (0.116)	0.877 (0.927)	17.220 (0.141)	7.677 (0.809)
z_{m_t}	0.037	0.989	3.910 (0.418)	4.793 (0.309)	31.794 (0.001)	8.274 (0.763)

B. Conditional variance-covariance structure

$$\boldsymbol{C} = \begin{bmatrix} \begin{matrix} 4.561 \\ (0.000) \end{matrix} & \begin{matrix} 0.319 \\ (0.251) \end{matrix} \\ & \begin{matrix} -0.117 \\ (0.162) \end{matrix} \end{bmatrix}; \boldsymbol{B} = \begin{bmatrix} \begin{matrix} 0.597 \\ (0.000) \end{matrix} & \begin{matrix} 0.080 \\ (0.021) \end{matrix} \\ \begin{matrix} -0.157 \\ (0.003) \end{matrix} & \begin{matrix} 0.896 \\ (0.000) \end{matrix} \end{bmatrix};$$

$$\boldsymbol{A} = \begin{bmatrix} \begin{matrix} -0.123 \\ (0.035) \end{matrix} & \begin{matrix} -0.010 \\ (0.742) \end{matrix} \\ \begin{matrix} 0.088 \\ (0.245) \end{matrix} & \begin{matrix} 0.393 \\ (0.000) \end{matrix} \end{bmatrix}; \boldsymbol{D} = \begin{bmatrix} \begin{matrix} -0.805 \\ (0.000) \end{matrix} & \begin{matrix} 0.229 \\ (0.000) \end{matrix} \\ \begin{matrix} -0.411 \\ (0.017) \end{matrix} & \begin{matrix} 0.218 \\ (0.113) \end{matrix} \end{bmatrix}.$$

Hypotheses testing

Diagonal VARMA	$H_0 : \gamma_{12}^{(i)} = \gamma_{21}^{(i)} = \theta_{12}^{(l)} = \theta_{21}^{(l)} = 0$, for $i, l = 1, 2$	0.000
No GARCH	$H_0 : \alpha_{ij} = \beta_{ij} = \delta_{ij} = 0$, for all i, j	0.000
No GARCH-M	$H_0 : \psi_{ij} = 0$, for all i, j	0.000
No asymmetry	$H_0 : \delta_{ij} = 0$, for $i, j = 1, 2$	0.000
Diagonal GARCH	$H_0 : \alpha_{12} = \alpha_{21} = \beta_{12} = \beta_{21} = \delta_{12} = \delta_{21} = 0$	0.002

Note : Sample period, monthly data: 1967:1-2011:3. Numbers in parentheses are tail areas of tests.

Table 7. The Bivariate Varma, Garch-in-Mean, Asymmetric BEKK Model With SUM MZM And The IP index

Equations (1) and (2) with $p = q = 2$ and $f = g = 1$

A. Conditional mean equation

$$\mathbf{a} = \begin{bmatrix} -1.229 \\ (0.607) \\ 2.717 \\ (0.000) \end{bmatrix}; \mathbf{\Gamma}_1 = \begin{bmatrix} 0.543 & -0.249 \\ (0.195) & (0.652) \\ 0.480 & 1.158 \\ (0.000) & (0.000) \end{bmatrix}; \mathbf{\Gamma}_2 = \begin{bmatrix} 0.306 & 0.254 \\ (0.220) & (0.580) \\ -0.467 & -0.210 \\ (0.000) & (0.494) \end{bmatrix};$$

$$\mathbf{\Theta}_1 = \begin{bmatrix} -0.333 & 0.267 \\ (0.400) & (0.643) \\ -0.532 & -0.417 \\ (0.000) & (0.240) \end{bmatrix}; \mathbf{\Theta}_2 = \begin{bmatrix} -0.286 & -0.104 \\ (0.103) & (0.209) \\ 0.296 & -0.258 \\ (0.001) & (0.207) \end{bmatrix}; \mathbf{\Psi}_1 = \begin{bmatrix} 0.323 & -0.154 \\ (0.147) & (0.744) \\ -0.299 & -0.037 \\ (0.001) & (0.564) \end{bmatrix}.$$

Residual diagnostics

	Mean	Variance	$Q(4)$	$Q^2(4)$	$Q(12)$	$Q^2(12)$
z_{y_t}	−0.016	1.018	5.153 (0.271)	1.101 (0.893)	12.883 (0.377)	8.695 (0.728)
z_{m_t}	−0.005	0.991	2.288 (0.682)	3.173 (0.529)	23.349 (0.024)	7.116 (0.849)

B. Conditional variance-covariance structure

$$\boldsymbol{C} = \begin{bmatrix} 4.625 & 0.906 \\ (0.000) & (0.486) \\ & 1.517 \\ & (0.456) \end{bmatrix}; \boldsymbol{B} = \begin{bmatrix} -0.603 & 0.050 \\ (0.000) & (0.803) \\ 0.198 & -0.754 \\ (0.349) & (0.053) \end{bmatrix};$$

$$\boldsymbol{A} = \begin{bmatrix} -0.027 & -0.062 \\ (0.564) & (0.527) \\ 0.168 & 0.572 \\ (0.031) & (0.000) \end{bmatrix}; \boldsymbol{D} = \begin{bmatrix} 0.743 & -0.049 \\ (0.000) & (0.672) \\ 0.344 & 0.256 \\ (0.079) & (0.623) \end{bmatrix}.$$

Hypotheses testing

Diagonal VARMA	$H_0 : \gamma_{12}^{(i)} = \gamma_{21}^{(i)} = \theta_{12}^{(l)} = \theta_{21}^{(l)} = 0$, for $i, l = 1, 2$	0.000
No GARCH	$H_0 : \alpha_{ij} = \beta_{ij} = \delta_{ij} = 0$, for all i, j	0.000
No GARCH-M	$H_0 : \psi_{ij} = 0$, for all i, j	0.000
No asymmetry	$H_0 : \delta_{ij} = 0$, for $i, j = 1, 2$	0.000
Diagonal GARCH	$H_0 : \alpha_{12} = \alpha_{21} = \beta_{12} = \beta_{21} = \delta_{12} = \delta_{21} = 0$	0.000

Note : Sample period, monthly data: 1967:1-2011:3. Numbers in parentheses are tail areas of tests.

Table 8. The Bivariate Varma, Garch-in-Mean, Asymmetric BEKK Model With Divisia MZM And The IP index

Equations (1) and (2) with $p = q = 2$ and $f = g = 1$

A. Conditional mean equation

$$\mathbf{a} = \begin{bmatrix} \begin{array}{c} 10.741 \\ (0.000) \\ \\ 14.767 \\ (0.000) \end{array} \end{bmatrix}; \mathbf{\Gamma}_1 = \begin{bmatrix} \begin{array}{cc} 2.949 & -2.671 \\ (0.000) & (0.000) \\ & \\ 1.799 & -1.734 \\ (0.000) & (0.000) \end{array} \end{bmatrix}; \mathbf{\Gamma}_2 = \begin{bmatrix} \begin{array}{cc} -2.119 & 2.043 \\ (0.000) & (0.000) \\ & \\ -1.967 & 1.876 \\ (0.000) & (0.000) \end{array} \end{bmatrix};$$

$$\mathbf{\Theta}_1 = \begin{bmatrix} \begin{array}{cc} -2.794 & 2.769 \\ (0.000) & (0.000) \\ & \\ -1.879 & 2.373 \\ (0.000) & (0.000) \end{array} \end{bmatrix}; \mathbf{\Theta}_2 = \begin{bmatrix} \begin{array}{cc} 1.577 & -0.615 \\ (0.000) & (0.000) \\ & \\ 1.475 & -0.545 \\ (0.000) & (0.004) \end{array} \end{bmatrix}; \mathbf{\Psi}_1 = \begin{bmatrix} \begin{array}{cc} 0.482 & -2.427 \\ (0.000) & (0.000) \\ & \\ 0.578 & -3.238 \\ (0.000) & (0.000) \end{array} \end{bmatrix}.$$

Residual diagnostics

	Mean	Variance	$Q(4)$	$Q^2(4)$	$Q(12)$	$Q^2(12)$
z_{y_t}	-0.028	0.994	10.226 (0.036)	1.047 (0.902)	25.881 (0.011)	5.684 (0.931)
z_{m_t}	0.028	0.965	3.290 (0.510)	0.944(0.918)	29.348 (0.003)	4.398 (0.975)

B. Conditional variance-covariance structure

$$\boldsymbol{C} = \begin{bmatrix} \begin{array}{cc} 4.356 & 0.052 \\ (0.000) & (0.869) \\ & \\ & 0.714 \\ & (0.381) \end{array} \end{bmatrix}; \boldsymbol{B} = \begin{bmatrix} \begin{array}{cc} 0.647 & 0.100 \\ (0.000) & (0.008) \\ & \\ -0.030 & 0.890 \\ (0.684) & (0.000) \end{array} \end{bmatrix};$$

$$\boldsymbol{A} = \begin{bmatrix} \begin{array}{cc} -0.207 & 0.007 \\ (0.009) & (0.752) \\ & \\ -0.075 & 0.300 \\ (0.399) & (0.000) \end{array} \end{bmatrix}; \boldsymbol{D} = \begin{bmatrix} \begin{array}{cc} 0.745 & -0.304 \\ (0.000) & (0.000) \\ & \\ 0.228 & -0.233 \\ (0.085) & (0.000) \end{array} \end{bmatrix}.$$

Hypotheses testing

Diagonal VARMA	$H_0 : \gamma_{12}^{(i)} = \gamma_{21}^{(i)} = \theta_{12}^{(l)} = \theta_{21}^{(l)} = 0$, for $i, l = 1, 2$	0.000
No GARCH	$H_0 : \alpha_{ij} = \beta_{ij} = \delta_{ij} = 0$, for all i, j	0.000
No GARCH-M	$H_0 : \psi_{ij} = 0$, for all i, j	0.000
No asymmetry	$H_0 : \delta_{ij} = 0$, for $i, j = 1, 2$	0.000
Diagonal GARCH	$H_0 : \alpha_{12} = \alpha_{21} = \beta_{12} = \beta_{21} = \delta_{12} = \delta_{21} = 0$	0.000

Note : Sample period, monthly data: 1967:1-2011:3. Numbers in parentheses are tail areas of tests.

Table 9. The Bivariate Varma, Garch-in-Mean, Asymmetric BEKK Model With SUM ALL And The IP index

Equations (1) and (2) with $p = q = 2$ and $f = g = 1$

A. Conditional mean equation

$$\mathbf{a} = \begin{bmatrix} \begin{matrix} 0.771 \\ (0.263) \end{matrix} \\ \begin{matrix} 0.029 \\ (0.980) \end{matrix} \end{bmatrix}; \boldsymbol{\Gamma}_1 = \begin{bmatrix} \begin{matrix} 1.014 \\ (0.002) \end{matrix} & \begin{matrix} -0.583 \\ (0.005) \end{matrix} \\ \begin{matrix} 0.640 \\ (0.369) \end{matrix} & \begin{matrix} 1.778 \\ (0.000) \end{matrix} \end{bmatrix}; \boldsymbol{\Gamma}_2 = \begin{bmatrix} \begin{matrix} -0.288 \\ (0.170) \end{matrix} & \begin{matrix} 0.525 \\ (0.006) \end{matrix} \\ \begin{matrix} -0.402 \\ (0.447) \end{matrix} & \begin{matrix} -0.847 \\ (0.000) \end{matrix} \end{bmatrix};$$

$$\boldsymbol{\Theta}_1 = \begin{bmatrix} \begin{matrix} -0.761 \\ (0.008) \end{matrix} & \begin{matrix} 0.448 \\ (0.208) \end{matrix} \\ \begin{matrix} -0.671 \\ (0.370) \end{matrix} & \begin{matrix} -1.154 \\ (0.000) \end{matrix} \end{bmatrix}; \boldsymbol{\Theta}_2 = \begin{bmatrix} \begin{matrix} 0.234 \\ (0.128) \end{matrix} & \begin{matrix} -0.061 \\ (0.669) \end{matrix} \\ \begin{matrix} 0.228 \\ (0.475) \end{matrix} & \begin{matrix} 0.328 \\ (0.164) \end{matrix} \end{bmatrix}; \boldsymbol{\Psi}_1 = \begin{bmatrix} \begin{matrix} 0.109 \\ (0.007) \end{matrix} & \begin{matrix} -0.117 \\ (0.420) \end{matrix} \\ \begin{matrix} -0.052 \\ (0.832) \end{matrix} & \begin{matrix} 0.063 \\ (0.159) \end{matrix} \end{bmatrix}.$$

Residual diagnostics

	Mean		Variance	$Q(4)$	$Q^2(4)$	$Q(12)$	$Q^2(12)$
z_{y_t}	−0.017	9	1.011	5.399 (0.248)	1.14 (0.886)	19.720 (0.072)	7.693 (0.808)
z_{m_t}	−0.027		1.004	8.544 (0.073)	1.094 (0.895)	39.855 (0.000)	16.205 (0.182)

B. Conditional variance-covariance structure

$$\boldsymbol{C} = \begin{bmatrix} \begin{matrix} 4.652 \\ (0.000) \end{matrix} & \begin{matrix} 0.646 \\ (0.102) \end{matrix} \\ & \begin{matrix} 1.932 \\ (0.000) \end{matrix} \end{bmatrix}; \boldsymbol{B} = \begin{bmatrix} \begin{matrix} -0.653 \\ (0.000) \end{matrix} & \begin{matrix} 0.084 \\ (0.288) \end{matrix} \\ \begin{matrix} 0.045 \\ (0.734) \end{matrix} & \begin{matrix} -0.242 \\ (0.019) \end{matrix} \end{bmatrix};$$

$$\boldsymbol{A} = \begin{bmatrix} \begin{matrix} 0.017 \\ (0.789) \end{matrix} & \begin{matrix} 0.073 \\ (0.019) \end{matrix} \\ \begin{matrix} -0.119 \\ (0.247) \end{matrix} & \begin{matrix} 0.773 \\ (0.000) \end{matrix} \end{bmatrix}; \boldsymbol{D} = \begin{bmatrix} \begin{matrix} 0.714 \\ (0.000) \end{matrix} & \begin{matrix} -0.025 \\ (0.562) \end{matrix} \\ \begin{matrix} 0.154 \\ (0.505) \end{matrix} & \begin{matrix} -0.658 \\ (0.010) \end{matrix} \end{bmatrix}.$$

Hypotheses testing

Diagonal VARMA	$H_0 : \gamma_{12}^{(i)} = \gamma_{21}^{(i)} = \theta_{12}^{(l)} = \theta_{21}^{(l)} = 0$, for $i, l = 1, 2$	0.000
No GARCH	$H_0 : \alpha_{ij} = \beta_{ij} = \delta_{ij} = 0$, for all i, j	0.000
No GARCH-M	$H_0 : \psi_{ij} = 0$, for all i, j	0.021
No asymmetry	$H_0 : \delta_{ij} = 0$, for $i, j = 1, 2$	0.000
Diagonal GARCH	$H_0 : \alpha_{12} = \alpha_{21} = \beta_{12} = \beta_{21} = \delta_{12} = \delta_{21} = 0$	0.012

Note : Sample period, monthly data: 1967:1-2011:3. Numbers in parentheses are tail areas of tests.

TABLE 10. THE BIVARIATE VARMA, GARCH-IN-MEAN, ASYMMETRIC BEKK MODEL WITH DIVISIA ALL AND THE IP INDEX

Equations (1) and (2) with $p = q = 2$ and $f = g = 1$

A. Conditional mean equation

$$\mathbf{a} = \begin{bmatrix} \underset{(0.000)}{-0.443} \\ \underset{(0.000)}{16.977} \end{bmatrix}; \mathbf{\Gamma}_1 = \begin{bmatrix} \underset{(0.000)}{1.271} & \underset{(0.000)}{0.139} \\ \underset{(0.000)}{17.081} & \underset{(0.698)}{0.072} \end{bmatrix}; \mathbf{\Gamma}_2 = \begin{bmatrix} \underset{(0.000)}{-0.381} & \underset{(0.000)}{-0.062} \\ \underset{(0.000)}{-13.232} & \underset{(0.000)}{-2.023} \end{bmatrix};$$

$$\mathbf{\Theta}_1 = \begin{bmatrix} \underset{(0.000)}{-1.056} & \underset{(0.000)}{-0.255} \\ \underset{(0.000)}{-17.080} & \underset{(0.006)}{0.526} \end{bmatrix}; \mathbf{\Theta}_2 = \begin{bmatrix} \underset{(0.000)}{0.270} & \underset{(0.000)}{0.122} \\ \underset{(0.000)}{9.527} & \underset{(0.000)}{4.288} \end{bmatrix}; \mathbf{\Psi}_1 = \begin{bmatrix} \underset{(0.483)}{-0.004} & \underset{(0.000)}{-0.098} \\ \underset{(0.004)}{0.456} & \underset{(0.000)}{-4.111} \end{bmatrix}.$$

Residual diagnostics

	Mean	Variance	$Q(4)$	$Q^2(4)$	$Q(12)$	$Q^2(12)$
z_{y_t}	−0.010	0.998	7.979 (0.092)	1.378 (0.847)	22.476 (0.032)	7.569 (0.817)
z_{m_t}	0.026	0.990	1.638 (0.801)	0.711 (0.949)	25.276 (0.025)	5.400 (0.943)

B. Conditional variance-covariance structure

$$\boldsymbol{C} = \begin{bmatrix} \underset{(0.000)}{4.314} & \underset{(0.499)}{0.132} \\ & \underset{(0.999)}{0.001} \end{bmatrix}; \boldsymbol{B} = \begin{bmatrix} \underset{(0.000)}{-0.700} & \underset{(0.000)}{-0.045} \\ \underset{(0.265)}{0.104} & \underset{(0.000)}{-0.936} \end{bmatrix};$$

$$\boldsymbol{A} = \begin{bmatrix} \underset{(0.957)}{0.008} & \underset{(0.611)}{-0.014} \\ \underset{(0.606)}{0.123} & \underset{(0.000)}{0.242} \end{bmatrix}; \boldsymbol{D} = \begin{bmatrix} \underset{(0.000)}{0.704} & \underset{(0.000)}{-0.171} \\ \underset{(0.425)}{0.179} & \underset{(0.008)}{-0.287} \end{bmatrix}.$$

Hypotheses testing

Diagonal VARMA	$H_0 : \gamma_{12}^{(i)} = \gamma_{21}^{(i)} = \theta_{12}^{(l)} = \theta_{21}^{(l)} = 0$, for $i, l = 1, 2$	0.000
No GARCH	$H_0 : \alpha_{ij} = \beta_{ij} = \delta_{ij} = 0$, for all i, j	0.000
No GARCH-M	$H_0 : \psi_{ij} = 0$, for all i, j	0.000
No asymmetry	$H_0 : \delta_{ij} = 0$, for $i, j = 1, 2$	0.000
Diagonal GARCH	$H_0 : \alpha_{12} = \alpha_{21} = \beta_{12} = \beta_{21} = \delta_{12} = \delta_{21} = 0$	0.000

Note : Sample period, monthly data: 1967:1-2011:3. Numbers in parentheses are tail areas of tests.