**Strengthening the Second Pillar:**

**A Greater Role for Money in Achieving the ECB's Nominal Objectives***

Michael T. Belongia
Department of Economics
University of Mississippi
Box 1848
University, MS 38677
mvpt@earthlink.net

and

Peter N. Ireland
Department of Economics
Boston College
140 Commonwealth Avenue
Chestnut Hill, MA 02467
peter.ireland@bc.edu

August 2020

**Abstract**:  Like most central banks, the European Central Bank makes and implements its monetary policy decisions by adjusting its targets for short-term interest rates in response to information gleaned from a wide range of macroeconomic indicators and projections.  Unlike many other central banks, however, the ECB also monitors money growth as a "cross check" against the macroeconomic analysis that guides its policies of interest rate management.  This paper argues that making further use of this "second pillar" would help the ECB to better achieve its nominal objectives in the present environment of exceptionally low inflation.  By modifying the "P-star" framework – a small-scale model with Quantity Theory foundations – the paper shows how the ECB could use its influence over Divisia money growth to stabilize nominal spending around a target path, even while its traditional interest rate policies are constrained by the zero lower bound.

**JEL codes**:  E58, E52, E51

**Strengthening the Second Pillar:**

**A Greater Role for Money in Achieving the ECB's Nominal Objectives**

Most central banks, including the European Central Bank (ECB), implement their monetary policy decisions by setting targets for three short-term interest rates. Adjustments to interest rate targets judged necessary to keep inflation close to the ECB's medium-term target – "below, but close to, two percent" – get made based on extensive analysis of a wide range of macroeconomic indicators and projections. The ECB, however, complements its interest rate strategy within a "two-pillar" approach that also monitors the behavior of the money supply. In particular, the behavior of money is meant to serve as a longer run "cross-check" on the accumulated influence of its interest rate decisions on the inflation objective.[1] Assigning this role to money is motivated by the standard Quantity Theory prediction that inflation ultimately is determined by the rate of growth in the money supply. The idea, therefore, is that monitoring money growth serves as a check on whether a series of short run decisions about settings for its interest rate target is moving the ECB away from achieving its goal of price stability. In this framework, money is not an intermediate target *per se,* but serves as a supporting indicator on the long-run thrust of ECB policy.

Recent events have placed great stress on the "first pillar" of ECB strategy, involving macroeconomic analysis and traditional interest rate management. As shown in the top panel of Figure 1, the global financial crisis of 2008-9 and Europe's own sovereign debt crisis that followed prompted the ECB to lower the policy rate on its deposit facility to just 0.25 percent in 2009, then all the way to zero in 2012.[2] Despite these exceptionally low interest rates, inflation in the Euro Area continued to run persistently – and at times significantly – below the Bank's two-percent target. The remaining panels of Figure 1 show similar patterns, regardless of

---

[1] Gerlach (2004) and Issing (2006) discuss the ECB policy strategy and the motivations for it in greater detail.

[2] Gerlach and Lewis (2014) review the literature on how the ECB traditionally implemented its policies based on a short term interest rate before and after the financial crisis of 2008-9 and estimate a reaction function for the ECB's approach to setting its interest rate target.

whether inflation is measured based on changes in the consumer price index – the ECB's own preference – or the GDP deflator – the measure used in the statistical analysis below. Figure 2, meanwhile, shows that rates of nominal and real GDP growth in the post-2008 period have failed to return consistently to levels experienced previously.

While, in theory, the zero interest rate on currency sets a lower bound on short-term nominal interest rates, the added convenience to banks of holding funds on deposit at the central bank instead of in the form of vault cash also appears, in practice, to have allowed policy rates to fall into modestly negative territory in many economies.[3] In fact, Figure 1 reveals that, in response to continued sluggish growth and inflation, the ECB lowered its deposit rate to -0.1 percent in 2014 and followed with a series of steps reducing it to -0.5 percent, where it stands today. Although these further cuts in policy rates may have provided some additional monetary stimulus, the overall impression one gets from the data shown in Figures 1 and 2 is that the ECB, like many other central banks around the world, has struggled greatly in recent years to achieve its nominal and real stabilization objectives while traditional interest rate policies have been constrained by the zero lower bound.

These observations lead naturally to the question of whether, by shifting more of its emphasis to the second pillar of its policy strategy, the ECB might have been able to exploit Quantity Theoretic links between money growth and either nominal spending or inflation to better achieve its stabilization goals for those variables. Figure 3 plots several Divisia monetary aggregates, originally constructed by Darvas (2014, 2015) and described in more detail below. Growth in all three of these aggregates is slower, on average, over the period since 2008 than before. By each measure, in fact, the Euro Area experienced repeated episodes featuring little or no money growth over this time span. While the interest rate target series from Figure 1 is consistent with the popular view that the ECB had done "whatever it takes" to facilitate the economy's recovery from the financial and sovereign debt crisis, the patterns in money growth

---

[3] Jackson (2015) surveys the international experience with negative policy rates. Rognlie (2016) and Belongia and Ireland (2020) extend popular models of money demand and the monetary business cycle to weaken the no-arbitrage condition imposed by the possibility of costless currency storage and thereby allow for negative nominal interest rates.

suggest, to the contrary, that monetary policy has been insufficiently accommodative or even perversely restrictive. From this perspective, it certainly seems possible that by making better use of its second pillar, and using money growth not only as a cross-check but as an intermediate target, the ECB could have conducted policy more effectively throughout this period.

To explore this possibility in more detail, this paper employs a framework outlined by Holbrook Working (1923) and used, with only minor modifications, by Hallman, et al. (1991) and Orphanides and Porter (2000) in the "P-Star" model.[4] Belongia and Ireland (2015) adapt the P-star model to address the question of whether it could be used to target nominal GDP (NGDP); Belongia and Ireland (2017) present evidence to show that a NGDP targeting approach based on the P-star model would have allowed the Federal Reserve to circumvent obstacles, similar to those faced by the ECB, associated with the zero lower interest rate bound encountered in the aftermath of the financial crisis in 2008.

Consistent with the logic behind the ECB's decision to monitor the behavior of money, the P-star model is based on the Equation of Exchange and is meant to allow a policymaker to evaluate whether a value for the money stock is consistent with a goal for long-run stability in the aggregate nominal price level or nominal spending. Drawing further on the same logic, the model also can be used to identify a numerical value for the money stock that, by accounting for slow-moving trends in monetary velocity, can serve as an intermediate target in achieving the central bank's nominal stabilization objectives.

Here, the P-star model is used in conjunction with newly-available Divisia monetary aggregates for the Euro Area to evaluate this strategy's usefulness relative to the ECB's current objectives for the price level (or inflation as its rate of change) as well as an alternative goal for

---

[4] The first footnote in Hallman et al. (1991) gives credit to Alan Greenspan for suggesting the usefulness of a small-scale statistical model, like the P-star model, linking the Federal Reserve's M2 monetary aggregate to longer-term price trends. Like the second pillar of the ECB's strategy, therefore, the paper by Hallman et al. (1991) serves as a reminder that monetary policy analyses based on the Quantity Theory have, at times, been of interest to policymakers at the world's leading central banks as well as to readers of top, general-interest economics journals.

nominal GDP. The use of Divisia aggregates is motivated by the erratic behavior of simple-sum data attributable to their construction and questions that measurement error raise about the robustness of some of the existing research that attempts to link money to output, nominal spending, or the price level in the Euro Area.[5]

The discussion below explains the basic analytics of Working's framework and then discusses how it can be applied to the goal of stabilizing the price level or the level of nominal GDP. This is followed by a set of results based on the same regressions used by Hallman et al. (1991) to examine whether movements in the Divisia monetary aggregate precede movements in the price level and nominal income. The P-star model then is used to derive paths for the monetary aggregates that are implied by this policy framework. Comparisons of these implied paths to the actual paths taken by the Divisia monetary aggregates allows judgments to be made about whether, zero or even negative policy rates notwithstanding, the ECB's monetary policies were "too accommodative," "too restrictive," or "on target" with a view towards achieving a goal for stabilizing the price level or the level of nominal spending over the period from 2001 through 2019.[6]

These results from the Euro Area are consistent with those obtained previously for the United States in Belongia and Ireland (2015, 2017) and highlight links between Divisia money and nominal spending that are stronger than those between Divisia money and the price level. The general conclusion, therefore, is that the ECB could build on its second pillar, using the P-star model as the basis for setting intermediate targets for Divisia money and thereby achieving

---

[5] For detailed discussion of measurement error in traditional simple-sum aggregates and the construction of Divisia indexes of monetary service flows see, e.g. Barnett (1980, 1982, 2012). In this context, it is interesting to note that Working (1923) devoted an appendix of his paper to an attempt to create an "Index for a Medium of Exchange." Even though he was writing long before the era of financial innovations and the payment of interest on checkable deposits, he intuited that different components of a monetary aggregate should be weighted differently. Belongia (1996) and Hendrickson (2014) offer examples of how inferences about the influence of money on aggregate activity depend crucially on this measurement issue.

[6] Although Taylor (1993) is known for his presentation of a policy rule based on setting a target for a short term interest rate, he noted (pp.209-210) that, given a smooth path for velocity and an estimate of potential output, the quantity of money would be linked to a target value for the price level.

its nominal stabilization objectives effectively even when interest rate policies are constrained by the zero lower bound. Pursuing this extension to the two-pillar approach, however, would likely require the ECB to either augment or replace its existing goal for inflation with one for nominal GDP growth instead.

In previous research, Altimari (2001), Gerlach (2004), Assenmacher-Wesche and Gerlach (2006, 2008), Kaufmann and Kugler (2008, 2009), and Hall, et al. (2009) find that variations in the quantity of money are important influences on the behavior of inflation within the Euro Area. Similarly, Karfakis (2013) provides evidence of links between Euro Area money growth and output. These papers support the popular, pre-crisis conception of the ECB's second pillar. They confirm, in particular, that monitoring money growth can provide a useful cross-check against macroeconomic forecasts generated from a wider range of nonmonetary data. Trecoci and Vega (2000), Gerlach and Svensson (2003), and Czudaj (2011) focus specifically on the application of the P-star model to ECB policy and find a role for money as an indicator variable. As a group, these studies draw on data from samples prior to the global financial crisis of 2008 and the extended period of extremely low policy rates that followed. The present analysis therefore can be viewed, in part, as extending these previous analyses by assessing the information content of Euro Area monetary aggregates over the most recent period when the constraints imposed by the zero lower interest rate bound might have made the ECB's second pillar even more useful from an operational perspective.

Previous studies that investigate the link between Divisia monetary aggregates and inflation in the Euro Area include Stracca (2004), Maki-Franti (2007), and Binner, et al. (2009).[7] Binner, et al. (2009) also conduct tests of weak separability as a condition for creating an aggregate and find that Divisia M2 and M3 satisfy this condition but Divisia M1 does not. Again, the data used in these studies generally predate the global financial crisis and the even more recent zero interest rate episode. In work most closely related to the present study,

---

[7] Papers investigating the link between Divisia aggregates and economic activity in individual European countries prior to formation of the Euro Zone can be found in Belongia and Binner (2000).

Darvas (2014, 2015) constructs the Euro Area Divisia monetary aggregates that also are employed here, and shows that they are useful in identifying the effects ECB policy had on output, prices, and interest rates over a sample period from 2001 through 2014 that includes the start of the zero interest rate episode. Likewise, Brill, et al. (2019) find Euro Area Divisia aggregates have predictive content for Euro Area recessions between 2003 and 2018. The present study complements their work, using estimates of the P-star model to show that Euro Area Divisia money might serve, usefully, as an intermediate target and not just as an information variable, even when the ECB's traditional interest rate policies are constrained by the zero lower bound.

**Working's Framework**

During the 1920s, both in the U.S. and Europe, a considerable amount of research was focused on the discovery of a monetary policy rule that would achieve price stability.[8] Much of the motivation for this work was found in Fisher's results (1923, 1925, 1926) that linked variations in the price level to fluctuations in output and employment, results that led Fisher to conclude the business cycle reflected a "dance of the dollar" rather than any inherent underlying cyclical tendencies of economic activity. In this spirit the proposed rules shared the intuition that a smooth path for the money supply would be likely to achieve the goal of price stability and, in so doing, dampen or eliminate variations in output and employment.

Working's (1923) contribution to this work was the derivation of a specific monetary policy rule that would identify a value for the money supply consistent with a goal of long-run

---

[8] Endres and Fleming (1998) discuss the broad research agenda on quantity-theoretic monetary policy rules in the 1920s, both in the United States and Europe. Hetzel (1985) and Humphrey (2001) provide background on the research agenda in the 1920s in the United States and the conflict between proposed rules of this type and the alternative policy strategy that eventually was adopted. McCallum (1990) applied his 1988 policy rule, which linked a path for the monetary base to nominal spending, to ask whether such a rule might have prevented the Great Depression.

price stability.[9]  In contrast to other work on this type of monetary policy rule, however, Working presented a method for dealing with lags between changes in money and prices.  In recognition of these lags the proposed policy framework embedded a desired long-run path for the price level and a long run path for money that would be consistent with achieving this goal.  Working's estimated trend value for the price level was determined by regressing the log value of the price level on time, time squared, and time cubed and, from that, extrapolating future values for the price level.  After establishing a trend for prices, a desired path for money could be determined and, from this, judgments about the relative stance of monetary policy could be made by examining a plot of the logarithm of the ratio of money to prices.  A quantity of money that implied a value for this ratio above the desired path and, hence, a rising price level, then would call for contractionary action; expansions in the money supply would be indicated when this ratio was below the target path.

The P-star model proposed by Hallman, et al. (1991) updated this earlier work by arranging terms in the Equation of Exchange as:

(1) $P^*_t = (M_t V^*_t)/Q^*_t$.

where $P^*_t$ is the long-run target value for the price level at time t, $M_t$ is the chosen measure of the money supply, $V^*_t$, is the long-run equilibrium value for velocity, and $Q^*_t$ is the value for potential real GDP at time t.[10]  In their original work, Hallman, et al. (1991), chose to represent trend velocity as a constant equal to the to the sample mean for the velocity of simple-sum M2.  As will be seen in what follows, the empirical failure of the P-Star model shortly after its publication can be attributed to this assumption of constant velocity.  With slow-moving trends in the velocities of simple sum monetary aggregates, any link between money and the price level in the long run would be tenuous.  Orphanides and Porter (2000), in recognition of this problem, revisited the earlier P-star model by estimating trend velocity values with a

---

[9] His proposal was based, in part, on an accumulating set of empirical tests of the Quantity Theory that had been reported since 1900. See Humphrey (1973) for a review of this work between 1900 and 1930.

[10] Humphrey (1989) discusses other precursors to the P-star model going back to Hume ([1752] 1977).

forecasting equation. They concluded the essential properties of the original P-star model are restored when trend velocity is allowed to vary over time. Therefore, if a central bank's goal is long run stability in the price level, it appears as if the P-star model might serve as reference framework if varying trends in velocity can be accommodated.

The same basic model was applied by Belongia and Ireland (2015) to the question of targeting nominal income through control of a monetary aggregate. This alternative goal required a rearrangement of the terms in equation (1) to express the Equation of Exchange as:

(2) $Y^*_t = M_t V^*_t$,

In this case a long-run target value for nominal GDP ($Y^*_t$) is associated with a value for a Divisia monetary aggregate ($M_t$) and an estimated value for that aggregate's trend velocity ($V^*_t$). In this work velocity was allowed to vary over time by estimating its trend value with the one-sided version of the Hodrick-Prescott (1997) filter described by Stock and Watson (1999).[11] Because the one-sided variant of the H-P filter only uses data up through period t and imposes the same setting $\lambda = 1600$ for the smoothing parameter used in the two-sided H-P filter, known values of the filter's terms allow the policy rule to be implemented in real time. The primary drawback of the one-sided filter is that using data only through time t produces a trend that is somewhat more volatile than what would result from use of the two-sided filter. Apart from estimating trend velocity by a different technique than that used by Orphanides and Porter (2000), however, equation (2) shares the same general features of equation (1). The two equations do differ, however, in one important respect: equation (2) does not require an independent estimate of potential output. Nonetheless, the relationship expressed in equation (2) suggests setting a target path for nominal spending ($Y^*_t$) and then making adjustments to the money supply in a manner that, over time, will direct the actual value for nominal income ($Y_t$), towards the target value ($Y^*_t$).

---

[11] Dueker (1993) addresses this same problem of variations in velocity for NGDP targeting by re-estimating McCallum's (1988) original model with a time-varying parameter estimation technique.

**The Data**

Before proceeding to the empirical work, a brief discussion of the data is in order. Constructing aggregate data for the Euro Area is complicated by at least two considerations. First, the composition of the Euro Area has changed as new countries have joined the European Union (EU): from an initial membership of twelve countries in 1999, the EU now is represented by twenty-seven countries. Only nineteen of these members, however, use the Euro as a common currency. Apart from the issue of abrupt shifts in the levels of aggregate data as new countries entered the EU, a second consideration is how the levels of output and prices for the individual member countries should be weighted to create single aggregate measures for the Euro Area. Currently, measures of output and the GDP deflator are published on the basis of the twelve early entrants to the EU as well as broader groups of members that include later entrants to the EU.

Darvas (2014) faced the same issues when constructing Divisia monetary aggregates for the Euro area.[12] As detailed in the Appendix to his paper, Darvas constructed measures of Divisia M1, M2 and M3 on the basis of a constant-composition of twelve countries and a changing-composition of seventeen countries. The aggregates based on the twelve initial countries were constructed to avoid the problem of abrupt shifts in their levels as EU membership expanded. In contrast, the aggregates based on seventeen countries potentially faced the problem of level shifts but had the advantage of being more coherent with the price level and expanding scope of output as new countries entered the EU. After recognizing that the countries that joined the EU between 2007 and 2014 represented only 1.5 percent of Euro Area M3, Darvas (2015) conducted his subsequent empirical work on the basis of the constant-composition monetary data that match compatible twelve-country measures of output and the price level. In what follows, we use the Divisia aggregates based on a constant composition of the original twelve members of the EU and match them with measures of output and prices

---

[12] Barnett (2003, 2007) addresses the issues associated with aggregating monetary data across heterogeneous countries. Work on the empirical properties of Divisia aggregates for the Euro area includes Wesche (1997), Reimers (2002) and Stracca (2004).

also based on a constant composition of twelve countries.  Data on all three of the Divisia

monetary aggregates were calculated by Darvas (2014) and are provided by Bruegel.[13]

The compositions of the three monetary aggregates are shown in Table 1.  As with the

Federal Reserve's monetary data, M1 represents a narrow transactions aggregate.  M2

broadens its coverage by adding categories of savings and time deposits typically held by

consumers.  M3 expands still further by adding to M2 accounts more associated with business

transactions.

**Table 1: ECB's definitions of euro area monetary aggregates**

|  | M1 | M2 | M3 |
|---|---|---|---|
| 1. Currency in circulation | X | X | X |
| 2. Overnight deposits | X | X | X |
| 3. Deposits with an agreed maturity up to 2 years |  | X | X |
| 4. Deposits redeemable at a period of notice up to 3 months |  | X | X |
| 5. Repurchase agreements |  |  | X |
| 6. Money market fund (MMF) shares/units |  |  | X |
| 7. Debt securities up to 2 years |  |  | X |

Measures of velocity were calculated for each of the three Divisia monetary aggregates above in

conjunction with nominal GDP for the Euro Area based on twelve countries.  The one-sided H-P

filter then was applied to these measures of velocity to generate the values of trend velocity

shown in Figure 4.  The figure reveals slow-moving trends for all three aggregates which, if not

accounted for, would be likely to undermine the links between the quantity of money and a

nominal goal variable such as the price level or nominal spending.[14]


**Evaluating the Performance of a Rule**

The important characteristic of any rule based on (1) or (2) is whether its guidance will

tend to return the goal variable back toward the target value if it has deviated from the target

path.  To examine this question in the original P-star model, Hallman et al. (1991) regressed

the change in inflation on four of its own lags and the lagged value of the price gap.  They

---

[13] http://bruegel.org/publications/datasets/divisia-monetary-aggregates-for-the-euro-area/

[14] Reynard (2007) finds that any attempt to discover long-run linkages between money and prices in the U.S., Switzerland, and the Euro Area must account for these slow moving trends in velocity.

defined the price gap as the difference between the natural logarithms of the target price level ($P^*_t$,) and the actual price level ($P_t$,). The target price level is determined by equation (1) where trend velocity ($V^*_t$) is represented by the values generated by the one-sided H-P filter, the quantity of money is one of the three Divisia aggregates discussed earlier, and potential output ($Q^*_t$) is introduced as a trend value estimated by the one-sided H-P filter. After taking logs, values for the target price level and the actual price level are represented in the equations below by $p^*_t$ and $p_t$. A positive and significant coefficient on the gap variable would indicate that inflation accelerates when the target value for the price level is greater than the actual value and, conversely, that inflation will tend to decline when the target value for the price level is below the actual value. A similar analysis will be applied later to nominal GDP by replacing the price gap with a comparable measure of the gap between the target and actual values of nominal income.[15]

Regressions of this form were estimated over a sample of quarterly data that spans 2001:1 through 2019:3. The detailed results for each of the three regressions, with absolute values of the $t$ statistics below each coefficient, are:

$$\Delta^2 p_t = \quad -0.911\Delta^2 p_{t-1} \quad -0.663\Delta^2 p_{t-2} \quad -0.327\Delta^2 p_{t-3} \quad +0.056\Delta^2 p_{t-4} \quad +0.044(p^*_{t-1}-p_{t-1})$$
$$\qquad\qquad (7.3) \qquad\qquad (4.1) \qquad\qquad (2.0) \qquad\qquad (0.5) \qquad\qquad (1.2)$$

for Divisia M1

$$\Delta^2 p_t = \quad -0.901\Delta^2 p_{t-1} \quad -0.654\Delta^2 p_{t-2} \quad -0.323\Delta^2 p_{t-3} \quad +0.057\Delta^2 p_{t-4} \quad +0.025(p^*_{t-1}-p_{t-1})$$
$$\qquad\qquad (7.1) \qquad\qquad (4.0) \qquad\qquad (2.0) \qquad\qquad (0.5) \qquad\qquad (0.4)$$

for Divisia M2 and

$$\Delta^2 p_t = \quad -0.901\Delta^2 p_{t-1} \quad -0.654\Delta^2 p_{t-2} \quad -0.323\Delta^2 p_{t-3} \quad +0.057\Delta^2 p_{t-4} \quad +0.005(p^*_{t-1}-p_{t-1})$$
$$\qquad\qquad (7.1) \qquad\qquad (4.0) \qquad\qquad (2.0) \qquad\qquad (0.5) \qquad\qquad (0.1)$$

---

[15] For these regressions we follow Hallman, et al. (1991) and multiply quarterly changes in the inflation rate and nominal GDP growth by 400 so that they are expressed in annualized percentage points. Also, as in Hallman, et al. (1991), values for the price level and nominal GDP gap variables are multiplied by 100 so that they are measured in percentage points. Each regression also included a constant term but, in the interest of space, these values are not shown.

for Divisia M3. In each case, the coefficient on the lagged price gap, while positive, is small and statistically insignificant. As such, these results provide only weak evidence that inflation accelerates when the gap is positive and decelerates when the gap is negative. This behavior implies the actual value of the GDP deflator will not converge, reliably and quickly, to the long-run target defined in (1). Although monetary policy strategies that employ the money supply typically raise questions about which of several alternative monetary aggregates might best serve this role, the results suggest that the failure of the P-star model does not depend on that choice. These results for a twelve-country Euro Area are in contrast to earlier work that has linked money to inflation in the Euro Area and, specifically, to applications of the P-star model to Euro Area data from samples predating the 2008-9 financial crisis and zero interest rate episode that followed. They are consistent, however, with Belongia and Ireland's (2017) finding that the links between measures of money and the aggregate price level weaken, as well, in very recent samples of U.S. data.

The same regression described above also can be applied to the case where the monetary policy target is nominal GDP rather than the price level. In this case the dependent variable is the change in the growth rate of nominal GDP and the gap variable is defined as the difference between the log of the target value for nominal GDP based on equation (2) and the log of the actual value of nominal GDP; these are represented by $y^*_{t-1}$ and $y_{t-1}$ below. The variable of interest is the lagged value of the nominal income gap and, specifically, whether its coefficient is positive and statistically significant. If so, as previously, the results would indicate that the growth rate of nominal GDP accelerates when the target value for nominal income is greater than the actual value and decelerates when the target value is below the actual value.

The results of this exercise are:

$$\Delta^2 y_t = \underset{(3.1)}{-0.356\Delta^2 y_{t-1}} \quad \underset{(1.8)}{-0.209\Delta^2 y_{t-2}} \quad \underset{(1.6)}{-0.183\Delta^2 y_{t-3}} \quad \underset{(0.3)}{-0.030\Delta^2 y_{t-4}} \quad \underset{(4.6)}{+0.450(y^*_{t-1} - y_{t-1})}$$

for Divisia M1

$$\Delta^2 y_t = \quad -0.246\Delta^2 y_{t-1} \quad -0.069\Delta^2 y_{t-2} \quad -0.038\Delta^2 y_{t-3} \quad -0.087\Delta^2 y_{t-4} \quad +0.623(y^*_{t-1} - y_{t-1})$$
$$\qquad\qquad (2.1) \qquad\qquad (0.6) \qquad\qquad (0.3) \qquad\qquad (0.7) \qquad\qquad (3.8)$$

for Divisia M2 and

$$\Delta^2 y_t = \quad -0.205\Delta^2 y_{t-1} \quad -0.040\Delta^2 y_{t-2} \quad -0.032\Delta^2 y_{t-3} \quad -0.079\Delta^2 y_{t-4} \quad +0.406(y^*_{t-1} - y_{t-1})$$
$$\qquad\qquad (2.2) \qquad\qquad (0.8) \qquad\qquad (0.5) \qquad\qquad (0.2) \qquad\qquad (2.8)$$

for Divisia M3.

In sharp contrast to the case of the price level, the results for nominal GDP indicate a large, positive, and significant coefficient for the lagged value of the nominal income gap. Once again, these results for the Euro Area echo those derived previously, for the U.S., in Belongia and Ireland (2017). They indicate that the Divisia monetary aggregates can serve as useful intermediate targets for monetary policy, even when interest rates are constrained by the zero lower bound, provided that nominal income replaces the price level in defining the central bank's stabilization objectives.

Some insights into the results are revealed by the observation that the price gap used in the first set of regressions and the nominal income gap used in the second set of regressions differ, algebraically, by the estimated output gap. With lower-case variables continuing to denote the natural logs of the corresponding upper-case variables, (1) and (2) imply that

$$(p^*_t - p_t) - (y^*_t - y_t) = (m_t + v^*_t - q^*_t - p_t) - (m_t + v^*_t - q_t - p_t) = q_t - q^*_t.$$

This decomposition suggests that slow convergence of the price gap compared to the nominal spending gap reflects, in turn, slow convergence of real GDP back to its natural, or trend, level.

Table 2 provides evidence consistent with this conjecture, by reporting the values of t-statistics from standard Dickey-Fuller tests applied to each of these variables. While spending gaps based on all three Divisia monetary aggregates are clearly mean-reverting, the price gap appears stationary only when measured with reference to Divisia M1. Meanwhile, the Dickey-Fuller test rejects its null hypothesis of a unit root in the output gap series, but the close proximity of the test statistic to its critical value indicates that the output gap, though

stationary as it should be, exhibits considerable persistence nevertheless, which then passes

through to the price gap as well.

In addition, if a lagged value of the output gap is added to the price gap equations

reported above, its coefficient is positive and significant in each case.[16]  Thus, the difference

between the two sets of regression results also may reflect the possibility that an adverse

supply shock under a price level or inflation target may lead to a monetary tightening whereas,

under a nominal income target, the effects of an adverse supply shock on the price level and

output would tend to offset each other and not give the same signal.  Taken together, these

results indicate that the P-star model works better when applied to nominal income instead of

the price level, in large part by sidestepping issues associated with output gap measurement.[17]

**Table 2.**  Dickey-Fuller Statistics for the Price, Spending, and Output Gap Variables

| Variable | ADF Statistic (5% critical value = -2.90) |
|---|---|
| Price gap based on Divisia M1 | -3.07 |
| Price gap based on Divisia M2 | -1.93 |
| Price gap based on Divisia M3 | -1.77 |
| Spending gap based on Divisia M1 | -4.30 |
| Spending gap based on Divisia M2 | -4.46 |
| Spending gap based on Divisia M3 | -4.13 |
| Output gap | -3.35 |

**Identifying a Target Path for Money**

---

[16] Assenmacher-Wesche and Gerlach (2008) find that high-frequency variations in the output gap help explain the behavior of inflation in the Euro Area.

[17] As noted by Orphanides and van Norden (2002), problems associated with estimated potential output and, hence, the output gap in real time also can undermine the practical usefulness of a Taylor (1993) rule.  See, for example, Beckworth and Hendrickson (2020) on this issue.

If a model of the P-star form is to be successful in achieving its nominal objective it must identify a specific path for a monetary aggregate that will move the goal variable to its target value. Because the regression results indicate the price level will not return to its long run target path within the P-star model, attention is directed to target values for the money supply when this framework is directed to targeting nominal income. To this end equation (2) can be re-arranged to isolate a path for the money supply that is consistent with an objective for nominal spending. This expression can be re-written as:

(3) $M^*_t = Y^T_t / V^*_t$

where $(Y^T_t)$ is the target value for nominal GDP and $V^*_t$ is the estimated trend velocity as in the previous section. The expression shows that, once trend values for velocity have been estimated and a target path for nominal spending has been established, values for the money supply consistent with this target can be calculated.

The top panel of Figure 5 plots the actual series for the logarithm of nominal GDP against a trend determined by fitting a least-squares regression on a constant and time trend over the eighteen-year period. The bottom panel, meanwhile, shows deviations of nominal spending from this trend. If the trend line in Figure 5 is interpreted as a target path for nominal GDP, equation (3) can be used to derive paths for the three alternative monetary aggregates that would be consistent with this nominal policy goal. Once the target paths for money implied by equation (3) have been calculated, money gaps – the difference between the actual value of a monetary aggregate and the target value implied by (3) – can be constructed.

These gaps, for each of the three monetary aggregates, are shown in Figure 6. They indicate considerable variation in the stance of monetary policy over the eighteen-year period. In general terms, Figure 6 indicates that monetary policy was restrictive before 2005, expansionary from 2005 through 2010, contractionary from 2011 through 2015, and roughly neutral from 2015 through the end of the sample. Because the regression results from above draw strong and significant links between movements in Divisia money and nominal GDP, these results also suggest that the ECB could have made better use of its second pillar once its policy rates hit the zero lower bound in 2012. If, at that time, the ECB had turned to money

growth as an intermediate target, it might have better insulated nominal spending and promoted a faster and more vigorous recovery from the financial and sovereign debt crises. Instead, the negative gaps implied by all three monetary aggregates indicate that, despite the persistence of low interest rates, the ECB's policies failed to deliver consistent monetary accommodation over that crucial period.

The one substantial disagreement among these monetary gaps is the alternative interpretation that is drawn from the behavior of the Divisia M1 gap against the gaps associated with Divisia M2 and M3. Whereas both of the broad aggregates indicate that monetary policy moved steadily from an expansionary to a contractionary stance from 2006 through 2009, the money gap based on Divisia M1 shows more volatility. According to this measure, monetary policy briefly became contractionary in the years leading up to the financial crisis of 2008, then abruptly reversed course by becoming expansionary immediately after.[18] A monetary cross-check using M1 might have served warning that, as argued by Hetzel (2016), the ECB's interest rate policies had become overly restrictive in 2007 and 2008.

The same monetarist tradition that rationalizes the ECB's two-pillar approach to monetary policymaking also emphasizes that, by imparting excess smoothness to interest rates over the business cycle, central banks often have generated pro-cyclical movements in the money supply that amplify, rather than ameliorate, output and price-level fluctuations triggered by non-monetary shocks.[19] Some evidence supporting this monetarist critique appears in the alternating patterns of monetary ease and restraint shown in Figure 6; additional evidence appears in the correlations reported in Table 3.

The first rows of the table indicate that gaps based on all three monetary aggregates share significant correlations, with the Divisia M2 and M3 gaps, in particular, moving in very close lock-step. Thus, while the choice between Divisia M1 and either of the two broader

---

[18] These wide swings in M1 growth just before, during, and after 2008 can also be seen in the raw data displayed in Figure 3.

[19] See Brunner and Meltzer (1964, p.35) for an early articulation of this view and Hetzel (2012, Ch.8) for a more recent exposition.

Divisia aggregates may be an issue deserving further consideration, the common signals sent by the Divisia aggregates as a group appear strong enough to make any one of them useful as an information variable or an intermediate target.[20]


**Table 3. Gap Correlations when Nominal Spending is the Policy Objective**

|  | M1 Gap | M2 Gap | M3 Gap | Price Deviation |
|---|---|---|---|---|
| M1 Gap | ------ |  |  |  |
| M2 Gap | 0.77 (10.4) | -------- |  |  |
| M3 Gap | 0.72 (8.8) | 0.97 (33.7) | ------- |  |
| Price Deviation | 0.25 (3.7) | 0.41 (3.9) | 0.32 (2.9) |  |
| NGDP Deviation | 0.41 (2.2) | 0.84 (13.1) | 0.79 (11.0) | 0.46 (4.4) |


The two bottom rows of Table 3 show that each of the money gaps also is positively correlated, as expected, with the deviations of nominal GDP from the target path shown in Figure 5 and a measure of deviations of the price level from a target path constructed in a similar manner.  The implication from these relationships is that the ECB's monetary policy, reflected in the money gaps, has been strongly expansionary at times when prices and nominal GDP are above target and strongly contractionary when prices and nominal GDP are below target.  These procyclical movements in money show how, consistent with the monetarist critique, a tendency for central bankers to smooth interest rates works, counterproductively, to make monetary policy a *source* of real and nominal volatility.  Moreover, the counterproductive effects of interest rate targeting may be exacerbated when interest rate policies are constrained by the zero lower bound.  Once more, these results suggest that, by placing greater emphasis on the second, monetary pillar of their policy strategy, ECB officials could pursue their stabilization goals more effectively, even after short-term interest rates reach their lower bound.

---

[20] If the issue of weak separability is considered as a condition for creating an aggregate measure of money, the results in Binner, et al. (2009) would offer one reason to reduce the choice to one between Divisia M2 and Divisia M3.

**Conclusion**

Monetary policy, when implemented by setting a target for a short-term interest rate, has been viewed to be impotent in recent years because interest rates have spent much of the time since 2009 at their lower bound near zero. This perceived impotence has led central banks to adopt a variety of innovative policy strategies including, among others, attempts to reduce longer-term interest rates through "forward guidance" – promises to keep short-term rates at exceptionally low levels even as real growth and inflation accelerate – and "quantitative easing" – large-scale purchases of government bonds and other assets.

Another option, discussed less frequently, is more in line with the monetary policy strategies of the late 1970s and early 1980s and focuses on linkages between the quantity of money and nominal magnitudes often regarded as variables a central bank might choose as goals for monetary policy. Using a Quantity-Theoretic framework suggested by Working (1923) and applying Barnett's (1980) economic approach to monetary aggregation, this paper illustrates how a central bank could derive a path for a Divisia monetary aggregate that would be consistent with the achievement of a long run nominal objective.[21] This framework appears fully consistent with the "second pillar" of the European Central Bank's existing monetary policy strategy, as it places significant weight on the information content of the monetary aggregates in discerning the stance of monetary policy and predicting its effects on the economy. The framework strengthens the second pillar, however, by outlining procedures through which appropriate intermediate targets for the Divisia monetary aggregates can be set in real time.

---

[21] Belongia and Ireland (2018, 2020) offer more examples of how a central bank might use the quantity of money to implement monetary policy when it faces the zero lower bound constraint on short term interest rates. These studies – one using a structural VAR and the other using a New Keynesian dynamic, stochastic, general equilibrium model – apply to the United States, but the new results presented here suggest that their conclusions will extend to the Euro Area as well.

The results suggest that a policy strategy of this type could be employed to stabilize nominal GDP around a target path.[22]  Implementing the approach would therefore require the ECB to augment or replace its existing, pre-announced target for inflation with one for nominal income.  But by leaning more heavily on its second, monetary pillar, the ECB could make clear that it can and will achieve its stabilization objectives, even when its traditional interest rate policies are constrained by the zero lower bound.

---

[22] Blot, et al. (2015), Hughes-Hallett (2015), and Lechthaler, et al. (2015) discuss whether nominal GDP targeting is an appropriate objective for monetary policy in the Euro Area.  Garin, et al. (2016), Honkapohja and Mitra (2014), and Sheedy (2014) find nominal GDP targeting dominates inflation targeting in various theoretical models.

**Figure 1. Euro-Area Interest Rates and Inflation**



ECB Deposit Facility Interest Rate



Euro Area Consumer Price Inflation Rate



Euro Area GDP Deflator Inflation Rate

The top panel shows the administered interest rate on the European Central Bank's deposit facility. The bottom two panels show year-over-year rates of inflation, based on the hamonized index of consumer prices and the GDP deflator.
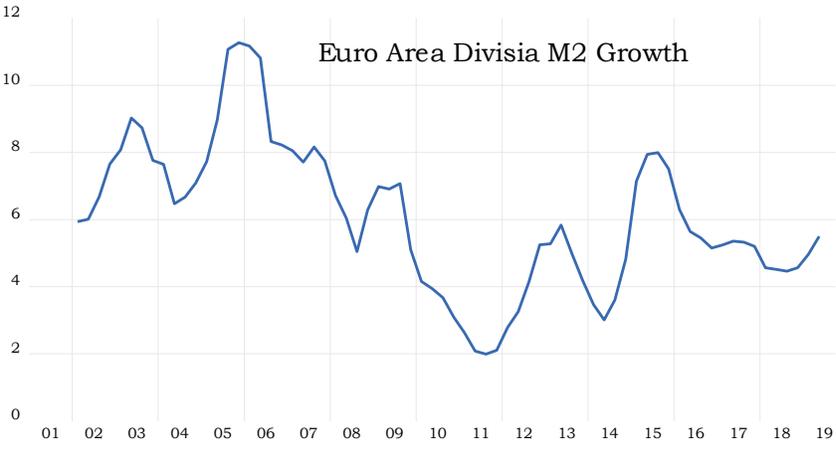
**Figure 2. Euro-Area Nominal and Real GDP Growth**


Euro Area Nominal GDP Growth


Euro Area Real GDP Growth
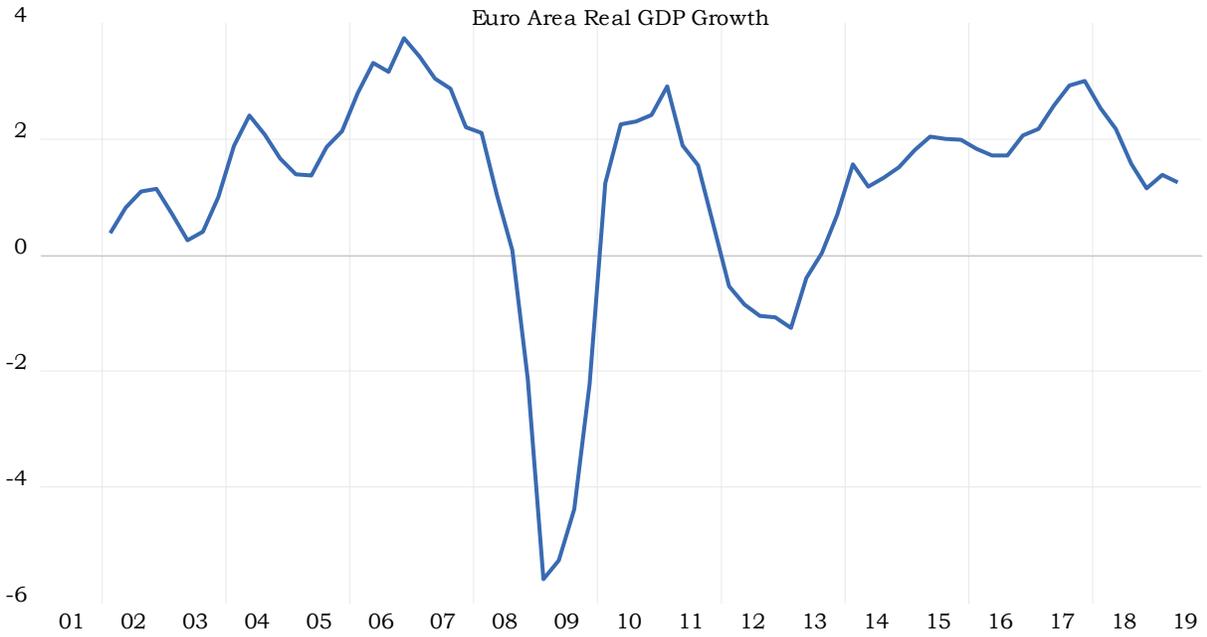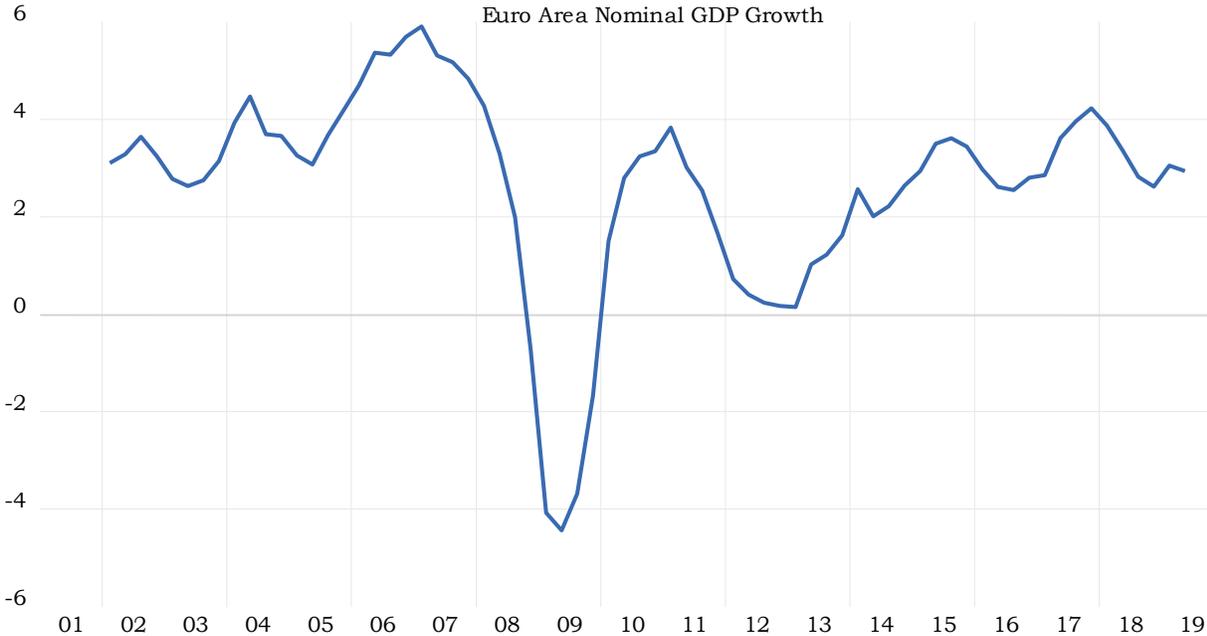
**Figure 3. Euro-Area Divisia Money Growth**

Euro Area Divisia M1 Growth

Euro Area Divisia M2 Growth

Euro Area Divisia M3 Growth
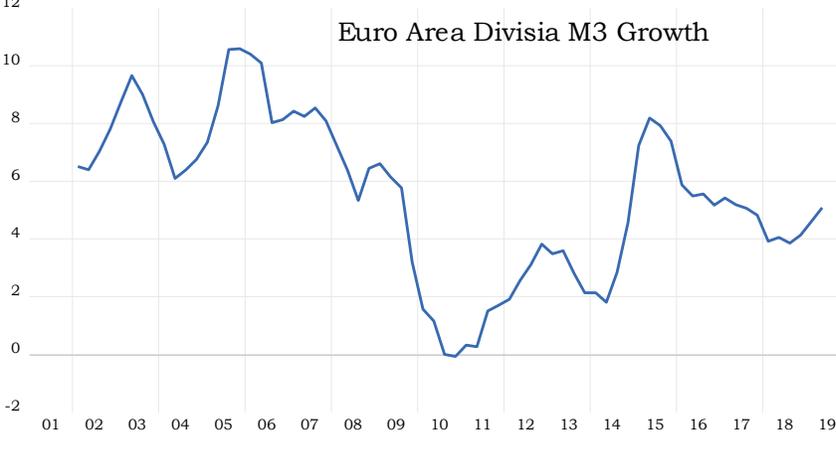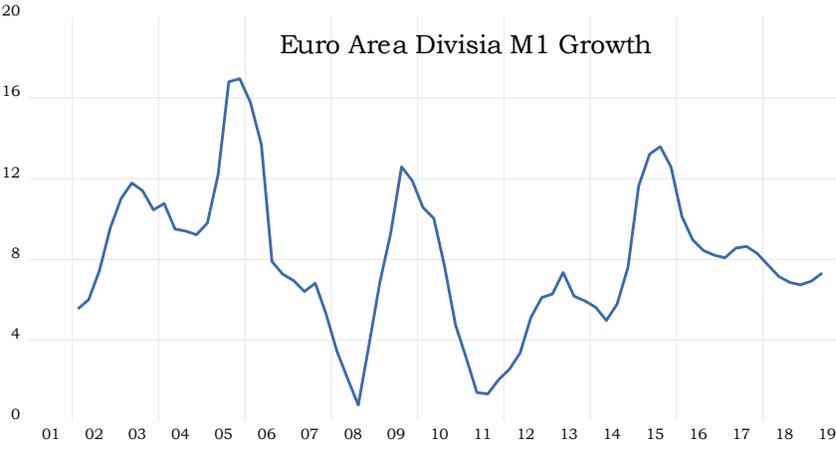
Panels plot the year-over-year growth rate of the Euro-Area Divisia monetary aggregates compiled by Darvas (2104).

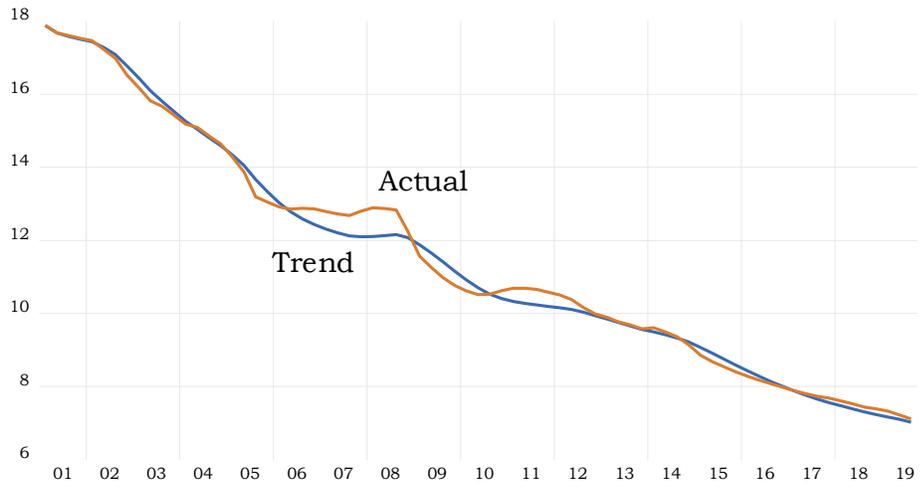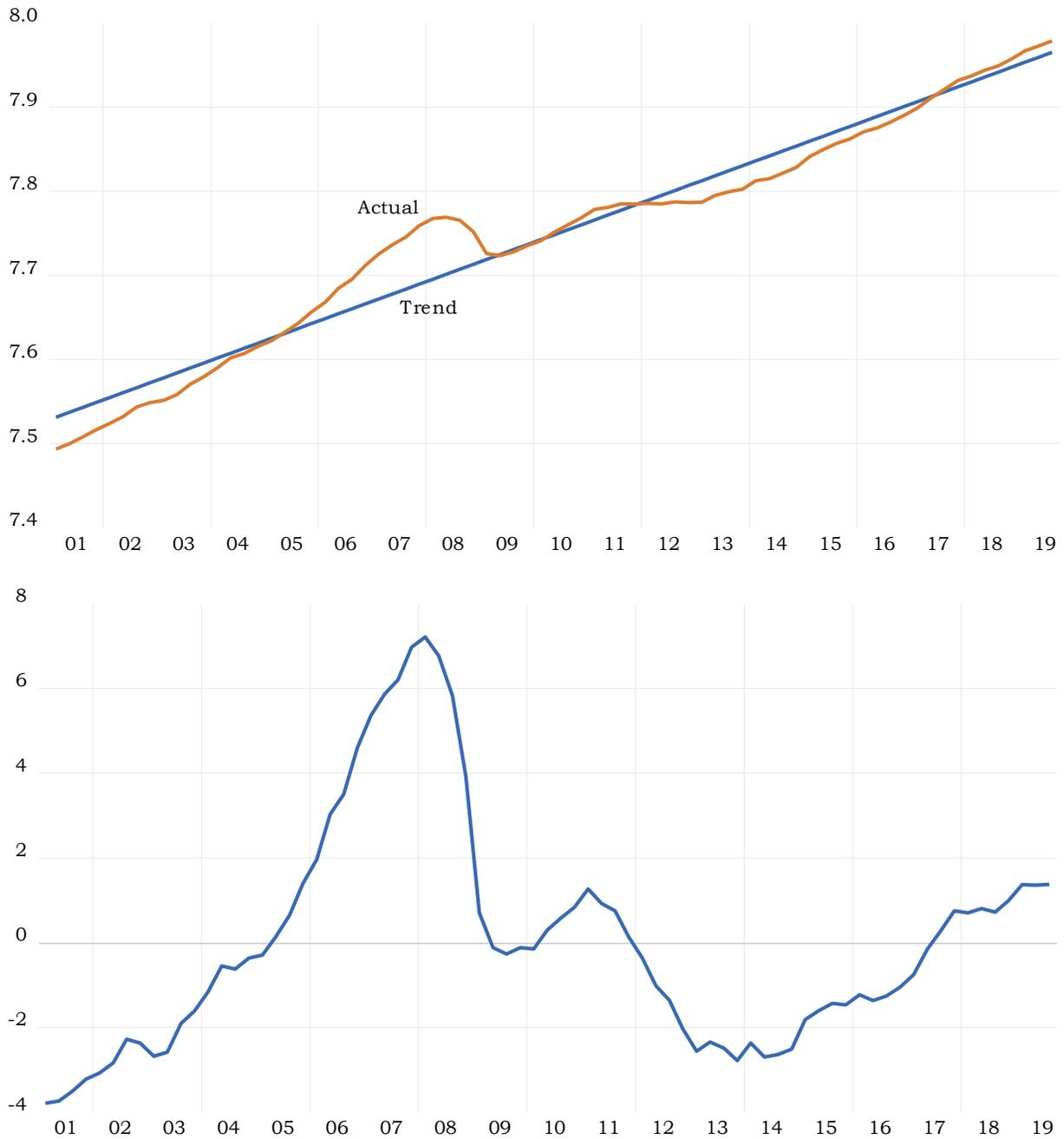**Figure 4. Actual and Trend Velocities for Divisia M1, M2, and M3**
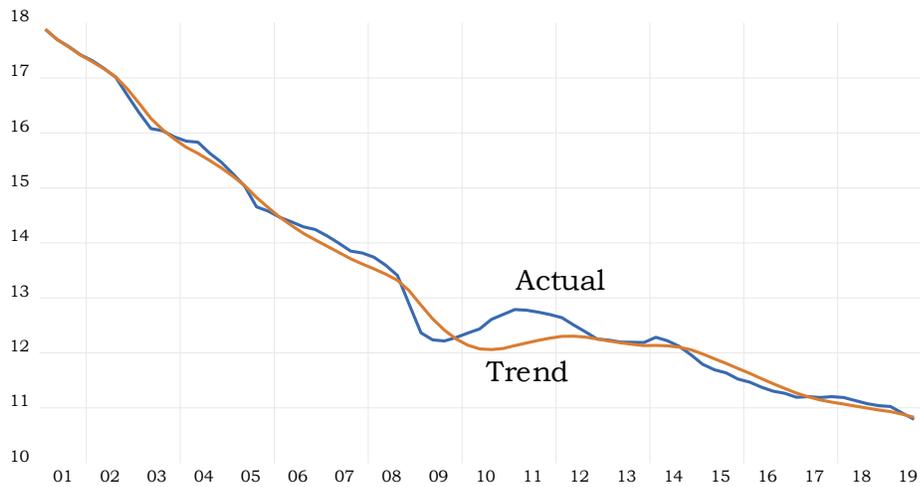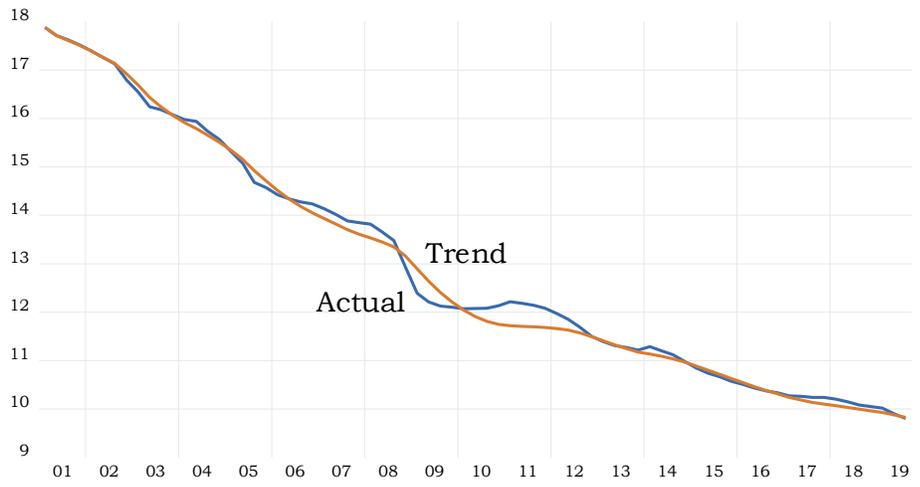
**Figure 5. Nominal GDP Relative to Trend**



The top panel compares the natural logarithm of nominal GDP to a trend line fitted by ordinary least squares to data from 2001:1 through 2015:4 and extrapolated through 2019:3. The bottom panel plots percentage-point deviations of actual nominal GDP from the estimated 2001-2019 trend.

**Figure 6. Gaps for Monetary Aggregates for a Nominal GDP Target (Divisia M1, M2, M3)**
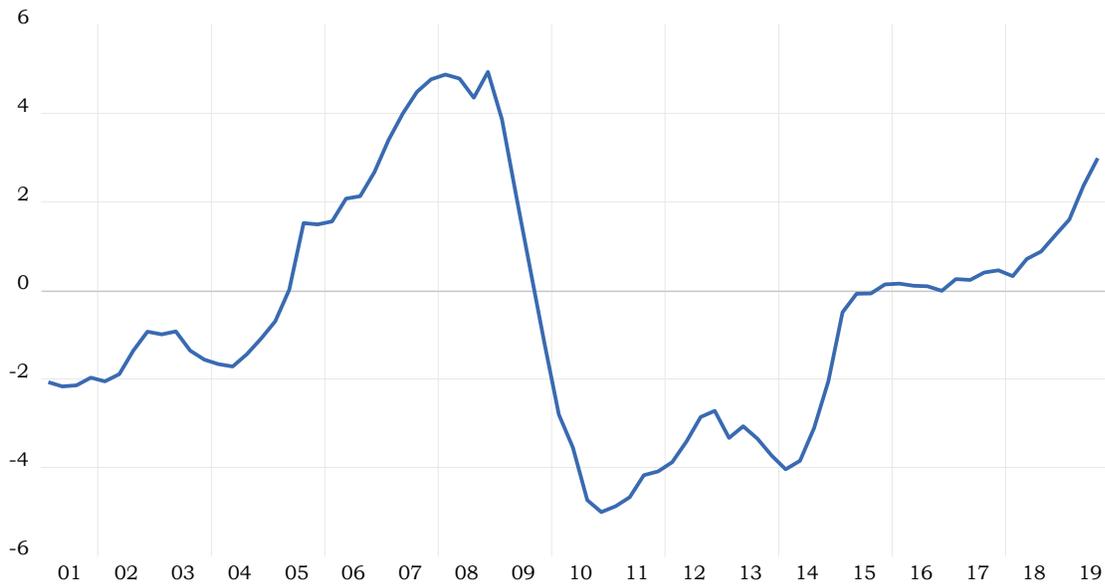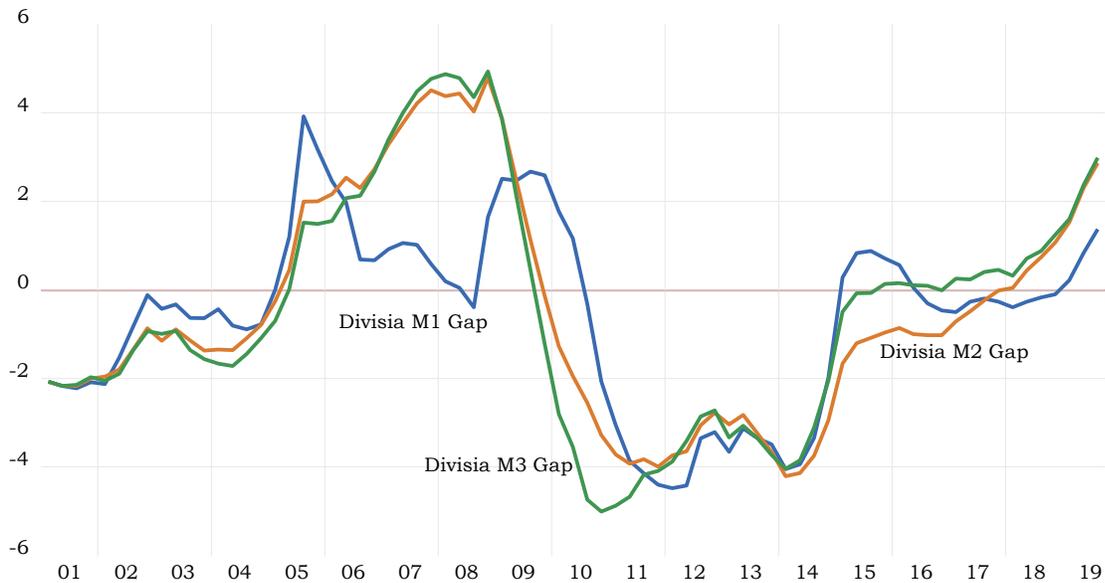
Divisia M1 Gap
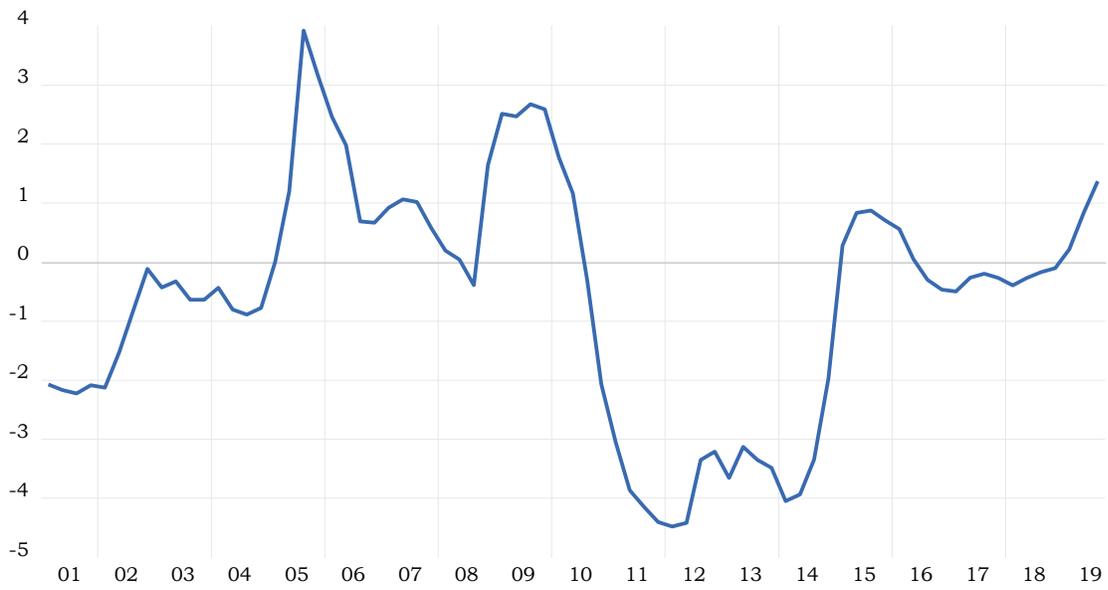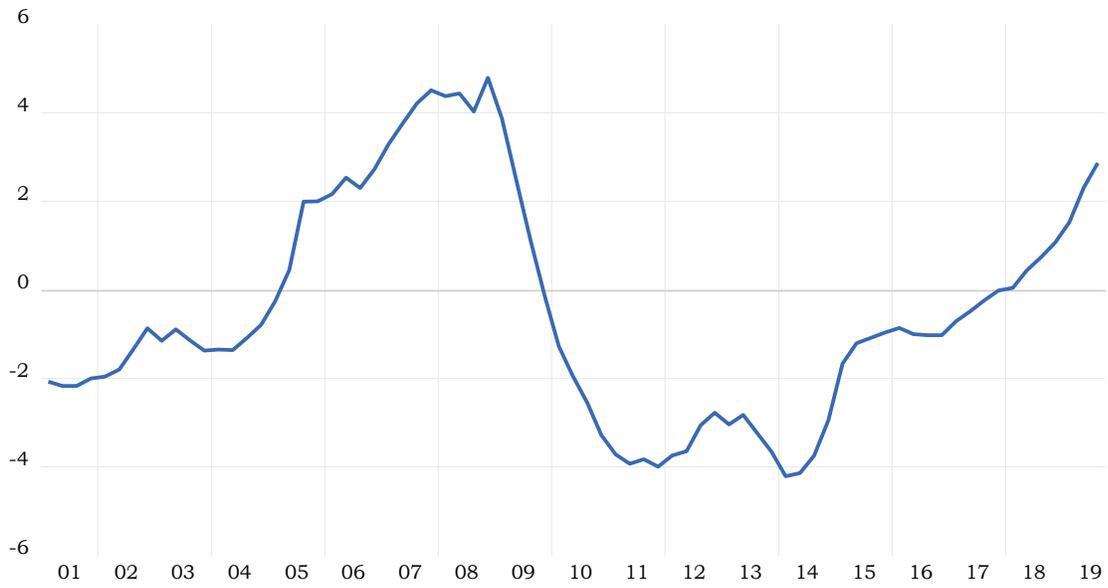


Divisia M2 Gap

Money Gaps for all Aggregates



The Divisia money gaps relative to a target for nominal GDP are measured as percentage-point differences between the actual value of each monetary aggregate and the desired value implied by equation (3).

# References

Altimari, S. Nicoletti. "Does Money Lead Inflation in the Euro Area?" European Central Bank Working Paper Series, Working Paper No. 63 (May 2001).

Assenmacher-Wesche, Katrin, and Stefan Gerlach. "Understanding the Link Between Money Growth and Inflation in the Euro Area," CEPR Discussion Paper No. 5683 (May 2006).

Assenmacher-Wesche, Katrin, and Stefan Gerlach. "Interpreting Euro Area Inflation at High and Low Frequencies," European Economic Review, 52 (August 2008): 964-986.

Barnett, William A. "Economic Monetary Aggregates: An Application of Index Number and Aggregation Theory," Journal of Econometrics vol. 14 (September 1980): 11-48. Reprinted in William Barnett and Apostolos Serleitis, The Theory of Monetary Aggregation. Amsterdam: Elsevier (2012): 11-48.

Barnett, William A. "The Optimal Level of Monetary Aggregation," Journal of Money, Credit, and Banking 14 (November 1982, Part 2): 687-710.

Barnett, William A. "Aggregation-Theoretic Monetary Aggregation over the Euro Area, when Countries are Heterogeneous," ECB Working Paper No. 260 (September 2003).

Barnett, William A. "Multilateral Aggregation-Theoretic Monetary Aggregation over Heterogeneous Countries," Journal of Econometrics 136 (February 2007): 457-482.

Barnett, William A. Getting It Wrong: How Faulty Monetary Statistics Undermine the Fed, the Financial System, and the Economy. Cambridge: MIT Press, 2012.

Beckworth, David and Joshua R. Hendrickson. "Nominal GDP Targeting and the Taylor Rule on an Even Playing Field," Journal of Money, Credit and Banking 52 (February 2020): 269-286.

Belongia, Michael T. "Measurement Matters: Recent Results from Monetary Economics Reexamined," Journal of Political Economy 104 (October 1996): 1065-1083.

Belongia, Michael T. and Jane M. Binner. Divisia Monetary Aggregates: Theory and Practice. New York: Palgrave (2000).

Belongia, Michael T. and Peter N. Ireland. "A 'Working' Solution to the Question of Nominal GDP Targeting," Macroeconomic Dynamics 19 (April 2015): 508-534.

Belongia, Michael T. and Peter N. Ireland. "Circumventing the Zero Lower Bound with Monetary Policy Rules Based on Money," Journal of Macroeconomics 54 (December 2017): 42-58.

Belongia, Michael T. and Peter N. Ireland. "Targeting Constant Money Growth at the Zero Lower Bound," International Journal of Central Banking 14 (March 2018): 159-204.

Belongia, Michael T. and Peter N. Ireland. "A Reconsideration of Money Growth Rules," Manuscript, Boston College (April 2020).

Binner, Jane M., Rakesh K. Bissoondeeal, Thomas C. Elger, Barry E. Jones, and Andrew W., Mullineux. "Admissible Monetary Aggregates for the Euro Area," Journal of International Money and Finance 28 (February 2009): 99-114.

Blot, Christophe, Jerome Creel, and Xavier Ragot.  "Flexible Inflation Targeting vs Nominal GDP Targeting in the Euro Area," in Is Nominal GDP Targeting a Suitable Tool for ECB Monetary Policy? European Parliament Monetary Dialogue (2015): 45-61.

Brill, Maximilian, Dieter Nautz, Dieter, and Lea Sieckmann. "Divisia Monetary Aggregates for a Heterogeneous Euro Area," Discussion Paper 9, Free University Berlin, School of Business and Economics (June 2019).

Brunner, Karl and Allan H. Meltzer. "An Alternative Approach to the Monetary Mechanism," Report to the Subcommittee on Domestic Finance, Committee on Banking and Currency, U.S. House of Representatives (August 1964).

Czudaj, Robert. "P-star in Times of Crisis – Forecasting Inflation in the Euro Area," Economic Systems 35 (September 2011): 390-407.

Darvas, Zsolt. "Does Money Matter in the Euro Area? Evidence from a new Divisia Index," Working Papers 1401, Department of Mathematical Economics and Economic Analysis, Corvinus University of Budapest (November 2014).

Darvas, Zsolt. "Does Money Matter in the Euro Area? Evidence from a new Divisia Index," Economics Letters 133 (2015): 123-126

Dueker, Michael J. "Can Nominal GDP Targeting Rules Stabilize the Economy?" Federal Reserve Bank of St. Louis Review 75 (May/June 1993): 15-29.

Endres, A.M. and Grant A. Fleming. "The Early Development of Monetary Policy Rules: The View from Geneva in the 1920s," Journal of Monetary Economics 42 (July 1998): 375-386.

Fisher, Irving. "The Business Cycle Largely a 'Dance of the Dollar,'" Journal of the American Statistical Association 18 (December 1923): 1024-1028.

Fisher, Irving. "Our Unstable Dollar and the So-Called Business Cycle," Journal of the American Statistical Association 20 (June 1925): 179-202.

Fisher, Irving. "A Statistical Relation between Unemployment and Price Changes," International Labour Review 13 (June 1926): 785-792.

Garín, Julio, Robert Lester, and Eric Sims. "On the Desirability of Nominal GDP Targeting," Journal of Economic Dynamics and Control 69 (August 2016): 21-44.

Gerlach, Stefan. "The Two Pillars of the European Central Bank," Economic Policy 19 (October 2004): 389-439.

Gerlach, Stefan and Lars E.O. Svensson. "Money and Inflation in the Euro Area:  A Case for Monetary Indicators?" Journal of Monetary Economics 50 (November 2003): 1649-1672.

Gerlach, Stefan and John Lewis. "Zero Lower Bound, ECB Interest Rate Policy and the Financial Crisis," Empirical Economics 46 (May 2014): 865-886.

Hall, Stephen G., George Hondroyiannis, P.A.V.B Swamy, and George S. Tavlas. "Assessing the Causal Relationship between Euro-Area Money and Prices in a Time-Varying Environment," Economic Modelling 26 (July 2009): 760-766.

Hallman, Jeffrey J., Richard D. Porter and David H. Small. "Is the Price Level Tied to the M2 Monetary Aggregate in the Long Run?" <u>American Economic Review</u> 81 (September 1991): 841-858.

Hendrickson, Joshua R. "Redundancy or Mismeasurement? A Reappraisal of Money," <u>Macroeconomic Dynamics</u> 18 (October 2014): 1437-1465.

Hetzel, Robert L. "The Rules versus Discretion Debate over Monetary Policy in the 1920s," Federal Reserve Bank of Richmond <u>Economic Review</u> (November/December 1985): 3-14.

Hetzel, Robert L. <u>The Great Recession: Market Failure or Policy Failure?</u> Cambridge: Cambridge University Press (2012).

Hetzel, Robert L. "What Caused the Great Recession in the Eurozone?" Working Paper 16-10, Federal Reserve Bank of Richmond (August 2016).

Hodrick, Robert J. and Edward C. Prescott. "Postwar U.S. Business Cycles: An Empirical Investigation," <u>Journal of Money, Credit, and Banking</u> 29 (February 1997): 1-16.

Honkapohja, Seppo and Kaushik Mitra. "Targeting Nominal GDP or Prices: Guidance and Expectation Dynamics," CEPR Discussion Paper No. 9857 (March 2014).

Hughes Hallett, Andrew. "Is Nominal GDP Targeting a Suitable Tool for ECB Monetary Policy?" in <u>Is Nominal GDP Targeting a Suitable Tool for ECB Monetary Policy?</u> European Parliament Monetary Dialogue (2015): 7-26.

Humphrey, Thomas M. "Empirical Tests of the Quantity Theory of Money in the United States, 1900 – 1930," <u>History of Political Economy</u> 5 (Fall 1973): 285-316.

Humphrey, Thomas M. "Precursors of the P-star Model," Federal Reserve Bank of Richmond <u>Economic Review</u> 75 (July 1989): 3-9.

Humphrey, Thomas M. "The Choice of a Monetary Policy Framework: Lessons from the 1920s," <u>Cato Journal</u> 21 (Fall 2001): 285-313.

Hume, David. "Of Money." Reprinted in *Essays: Moral, Political, and Literary*, 1777 Edition. Indianapolis: Liberty Fund ([1752] 1987): 281-294.

Issing, Otmar. "The ECB's Monetary Policy Strategy: Why Did We Choose a Two-Pillar Approach?" Address at the Fourth ECB Central Banking Conference, (November 10, 2006).

Jackson, Harriet. "The International Experience with Negative Policy Rates," Staff Discussion Paper 2015-1, Bank of Canada (November 2015).

Karfakis, Costas. "On Money and Output in the Euro Area: Is Money Redundant?" <u>International Economic Journal</u> 27 (September 2013): 487-496.

Kaufmann, Sylvia, and Peter Kugler. "Does Money Matter for Inflation in the Euro Area?" <u>Contemporary Economic Policy</u> 26 (February 2008): 590-606.

Kaufmann, Sylvia, and Peter Kugler. "A Monetary Real-Time Conditional Forecast of Euro Area Inflation," <u>Journal of Forecasting</u> 29 (January 2009): 388-405.

Lechthaler, Wolfgang and Mewael Tesfaselassie. "Is Nominal GDP Targeting a Suitable Tool for ECB Monetary Policy?" in Is Nominal GDP Targeting a Suitable Tool for ECB Monetary Policy? European Parliament Monetary Dialogue (2015): 27-44.

Maki-Franti, Petri. "The Information Content of the Divisia Monetary Aggregates in Forecasting Inflation in the Euro Area," Empirical Economics 33 (July 2006): 151-176.

McCallum, Bennett T. "Robustness Properties of a Rule for Monetary Policy," Carnegie-Rochester Conference Series on Public Policy 29 (January 1988): 173-203.

McCallum, Bennett T. "Could a Monetary Base Rule Have Prevented the Great Depression?" Journal of Monetary Economics 26 (August 1990): 3-26.

Orphanides, Athanasios, and Richard Porter. "P* Revisited: Money-Based Inflation Forecasts with a Changing Equilibrium Velocity," Journal of Economics and Business 52 (January-April 2000): 87-100.

Orphanides, Athanasios and Simon van Norden. "The Unreliability of Output-Gap Estimates in Real Time," The Review of Economics and Statistics 84 (November 2002): 569-583.

Reimers, Hans-Eggert. "Analysing Divisia Aggregates for the Euro Area," Discussion Paper 13/02, Economic Research Centre of the Deutsche Bundesbank (May 2002).

Reynard, Samuel. "Maintaining Low Inflation: Money, Interest Rates, and Policy Stance." Journal of Monetary Economics 54 (July 2007): 1441–1471.

Rognlie, Matthew. "What Lower Bound? Monetary Policy with Negative Interest Rates." Manuscript, Northwestern University, July 2016.

Sheedy, Kevin D. "Debt and Incomplete Financial Markets: A Case for Nominal GDP Targeting," Brookings Papers on Economic Activity (Spring 2014): 301-361.

Stracca, Livio. "Does Liquidity Matter? Properties of a Divisia Monetary Aggregate in the Euro Area," Oxford Bulletin of Economics and Statistics 66 (July 2004): 309-331

Taylor, John B. "Discretion versus Policy Rules in Practice," Carnegie-Rochester Series on Public Policy 39 (1993): 195-214.

Trecroci, Carmine and Juan Vega. "The Information Content of M3 for Future Inflation in the Euro Area," Review of World Economics (Weltwirtschaftliches Archiv) 138 (March 2002): 22-53.

Wesche, Katrin. "The Demand for Divisia Money in a Core Monetary Union," Federal Reserve Bank of St. Louis Review 79 (September/October 1997): 51-60.

Working, Holbrook. "Prices and the Quantity of Circulating Medium, 1890-1921," Quarterly Journal of Economics 37 (February 1923): 229-253.