APPLYING COMPLEXITY THEORY TO INTEREST RATES: EVIDENCE OF CRITICAL TRANSITIONS IN THE EURO AREA

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Views expressed are those of the author and do not necessarily reflect the position of DNB or the Eurosystem
Motivation

• Applying complexity theory to financial markets
  • assuming the financial system behaves like an eco-system

• Detect critical transitions in interest rates
  • money market (floor system)
  • bond market (safety trap)

• Link those regime changes to excess liquidity
Complexity theory

(A) Original stability landscape
Complexity indicators (1)

- Literature on complex systems identifies key early warning indicators (eg Scheffer et al., 2009)

- Critical slowdown
  \[ \tau = \frac{\ln(2)}{h}, \quad \text{with} \quad h = \frac{-t \ln(2)}{\ln\left(\frac{N_t}{N_0}\right)} \]

- Autocorrelation
  \[ N_t = c + \rho N_{t-1} + \epsilon_t \]
Complexity indicators (2)

- Variance
  \[ \sigma^2 = \frac{1}{n-1} \sum_{t=1}^{n} (N_t - \mu)^2 \]

- Skewness
  \[ \gamma = \frac{m_3}{\sigma^3} = \frac{\frac{1}{n} \sum_{t=1}^{n} (N_t - \mu)^3}{\left[ \frac{1}{n-1} \sum_{t=1}^{n} (N_t - \mu)^2 \right]^{3/2}} \]

- Flickering
  \[ \beta = \frac{\gamma^2 + 1}{\kappa} \]
Literature

- Dynamics of system near tipping point have generic properties (Scheffer et al., 2009)

- Guttal et al. (2016) and Diks et al. (2015) use complexity indicators to detect critical transitions in equity prices

- Quax et al. (2013) find early warning properties in interest rates swaps, prior to collapse of Lehman in 2008

- These studies use complexity indicators to detect financial crises, without explicitly linking these to changes in external (market) conditions
Approach

- Increased excess liquidity is change in external conditions that has fundamentally altered the financial system
Market functioning

Trading volume interbank market

Note. Trading volume in euro area unsecured interbank market. EUR billion, monthly averages.

Issuance of bonds by banks

Note. Net issuance of bonds by banks in the euro area (new issuance minus redemptions). EUR millions.
Risk of critical transition

• Substitution private market activities with central bank liquidity has weakened the resilience of financial system

• Market functioning depends on excess liquidity and so have become more vulnerable to (small) shocks, e.g. adjustment of monetary policy

• Feedback effects, following from reactions by investors, may exacerbate shock effects

• Interest rates resemble the state variable, indicating both the gradual change of the system before bifurcation and the critical transition beyond that point
Floor system in money market

Money market rates

EONIA margin and excess liquidity

Excess liquidity / Eonia volume
Safety trap in bond market

Yield on safe assets

AAA rate margin and excess liquidity

Excess liquidity / Eonia volume
Markov switching

- \( y_t = c(s_t) + \varphi y_{t-1} + \varepsilon_t \)
- \( s_t = 1, 2 \)
- \( p_{ij} = p(s_t = j \mid s_{t-1} = i) \) for any \( i, j = 1, 2 \) (transition probability)
## Estimation output for money market
Markov switching regression (2 regimes, switching in mean)
Dependent variable is difference between EONIA rate and deposit rate (DFR)
sample period 2006-2010 (daily observations)

<table>
<thead>
<tr>
<th>EONIA - DFR</th>
<th>Coeff.</th>
<th>sign.</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant (1)</td>
<td>0.48</td>
<td>***</td>
</tr>
<tr>
<td>constant (2)</td>
<td>0.80</td>
<td>***</td>
</tr>
<tr>
<td>EONIA - DFR_{t-1}</td>
<td>0.88</td>
<td>***</td>
</tr>
<tr>
<td>EONIA - DFR_{t-2}</td>
<td>0.11</td>
<td>***</td>
</tr>
</tbody>
</table>

Excess liquidity\(^1\)

| P_{11} | 0.94 |
| P_{22} | 0.95 |
| number of obs. | 1074 |

\(^1\) Excess liquidity is 10 days moving average of ratio excess liquidity / EONIA trading volume

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## Estimation output for safe asset market
Markov switching regression (2 regimes, switching in mean)
Dependent variable is difference between AAA govt bond rate and deposit rate (DFR)
sample period 2011-2016 (daily observations)

<table>
<thead>
<tr>
<th>AAA - DFR</th>
<th>Coeff.</th>
<th>sign.</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant (1)</td>
<td>-0.23</td>
<td>***</td>
</tr>
<tr>
<td>constant (2)</td>
<td>-0.09</td>
<td>***</td>
</tr>
<tr>
<td>AAA - DFR_{t-2}</td>
<td>0.86</td>
<td>***</td>
</tr>
<tr>
<td>AAA - DFR_{t-3}</td>
<td>0.12</td>
<td>***</td>
</tr>
</tbody>
</table>

Excess liquidity\(^1\)

| P_{11} | 0.98 |
| P_{22} | 0.99 |
| number of obs. | 1417 |

\(^1\) Excess liquidity is 10 days moving average of ratio excess liquidity / EONIA trading volume

### Notes
- ****, ***, * denote p-values less than or equal to 1%, 5%, 10%, respectively.
Dating the regime switches

- **Money market**: \[ \text{EONIA} - \text{DFR} \] < 10 bp
- **Bond market**: \[ \text{DFR} - \text{AAA rate} \] < 0 bp
Early warning signals

• 1\textsuperscript{st} moment of indicator: level exceeds 5% tail of distribution 100 days preceding critical transition

• 2\textsuperscript{nd} moment of indicator: significant trend 100 days before critical transition
  • Kendall rank correlation (K-r), measuring the concordance between indicator and time
  • Strong downward trend if K-r \approx -1; upward trend if K-r \approx +1
Signals of slowdown

- Significant slowdown according to level of indicator
- Slowdown in trend indicator in money market
Signals of autocorrelation

- Both the 1\textsuperscript{st} and 2\textsuperscript{nd} moments of indicator emit significant signals
Signals of variance

- Mixed signals for money and safe asset markets

Variance money market

Variance safe asset market

K-r = -0.68
p = 0.00

K-r = 0.63
p = 0.00
Signals of skewness and flickering

Skewness money market

Skewness safe asset market

Flickering money market

Flickering safe asset market
Quality of signals (1)

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Slowdown*</th>
<th>Autocorr</th>
<th>Variance</th>
<th>Skewness</th>
<th>Flickering</th>
</tr>
</thead>
<tbody>
<tr>
<td>EONIA rate – DFR</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type 1 error</td>
<td>0.87</td>
<td>0.95</td>
<td>0.97</td>
<td>0.93</td>
<td>0.84</td>
</tr>
<tr>
<td>Type 2 error</td>
<td>0.08</td>
<td>0.05</td>
<td>0.06</td>
<td>0.04</td>
<td>0.00</td>
</tr>
<tr>
<td>AAA rate – DFR</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type 1 error</td>
<td>0.84</td>
<td>0.85</td>
<td>0.83</td>
<td>0.87</td>
<td>0.87</td>
</tr>
<tr>
<td>Type 2 error</td>
<td>0.07</td>
<td>0.01</td>
<td>0.01</td>
<td>0.02</td>
<td>0.02</td>
</tr>
</tbody>
</table>

* For the slowdown indicator a low value implies a relevant early warning signal; for the other indicators a high value implies a relevant early warning signal (relevant in the sense that they are issued within a horizon of 100 days before a tipping point).

Note: Critical transition in money market assumed if (EONIA – DFR) < 10 bp. Critical transition in safe asset market assumed if (AAA rate – DFR) < 0. Type 1 error: (number of missed signals 100 days before critical transition / total number of critical transitions). Type 2 error: (number of false signals 100 days before critical transition / days without critical transition), taking into account the full sample period 2005-2016.
Quality of signals (2)

A. Switch to floor system in money market

Slowdown (decay)  Autocorrelation  Variance  Skewness  Flickering

B. Switch to safety trap bond market

Slowdown (decay)  Autocorrelation  Variance  Skewness  Flickering

Note: Area under the receiver operating characteristic curve (AUROC) along y-axis (red line), Horizon: days before tipping point in money market (1 July 2009). Dashed lines: 95% confidence intervals.

Note: Area under the receiver operating characteristic curve (AUROC) along y-axis (red line), Horizon: days before tipping point in safe asset market (11 March 2015). Dashed lines: 95% confidence intervals.
Link to excess liquidity (1)
Link to excess liquidity (2)

Skewness money market

Skewness safe asset market

Flickering money market

Flickering safe asset market
Some issues

- Indicators of natural systems applicable to economic, financial systems?
  - no rules of nature, but behavior as driving factor
  - less an issue for ex-post analyses

- Complicated to distinguish intended from unintended effects of monetary policy
  - new state endogenous on interventions by the central bank
  - new state notoriously hard to end
Conclusion

• Complexity theory provides useful framework to describe critical transitions in interest rates
• Indicators signal shift to corridor system in money market and safety trap in bond market
• Evidence for link between indicators and excess liquidity
• Insights can help central banks to strike a balance between supporting the financial system by injecting liquidity and unintended side-effects on market functioning