Consumer Preferences, the Demand for Divisia Money, and the Welfare Costs of Inflation*

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Abstract:

This paper uses neoclassical demand theory to calculate the welfare costs of inflation. It considers the demand interactions between money, consumption goods, and leisure, relaxes the assumption of fixed consumer preferences, and addresses the inter-related problems of estimation of money demand functions, instability of money demand relations, and monetary aggregation. It makes full use of the relevant economic theory and econometrics and generates inference in terms of long-run welfare costs of inflation that is internally consistent with the data and models used.

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1 Introduction

The recognition that the primary goal of monetary policy should be price stability has led to inflation targeting in advanced economies. Moreover, a large number of emerging and developing economies have switched from exchange rate targeting to inflation targeting, and many other countries are moving toward this monetary policy strategy. Most inflation targeting central banks adopted a 2% inflation target, but during the coronavirus pandemic, the Federal Reserve in the United States switched to a new monetary policy strategy that involves targeting an average inflation rate of 2%. After the adoption of inflation targets, inflation rates declined in most countries around the world. However, in the aftermath of the coronavirus pandemic, inflation rose above 8% in the United States and much higher in many advanced and emerging market economies. This has led to renewed interest on the welfare costs of inflation.

One method for calculating the welfare cost of inflation is based on Bailey (1956). In this approach, the welfare cost of inflation is defined as the change in the area under the inverse money demand curve corresponding to the change in the holdings of real money balances. Based on this approach, there are many estimates of the welfare costs of inflation in the United States. Lucas (2000) reports a welfare cost of inflation of about 1% of real income per year if the annual inflation rate is 10%. However, Ireland (2009) estimates the welfare cost of inflation to be around 0.23% of real income per year (if the annual inflation rate is 10%), which is significantly lower than Lucas's (2000) estimate. Also, Mogliani and Urga (2018) estimate a substantially lower welfare cost of inflation after 1976, Dai and Serletis (2019) find that the welfare cost of inflation declined significantly (by close to 50%) after the 1980s, and Miller *et al.* (2019) report estimates in the range of $0.025 - 0.75\%$ with an average of 0.27%, implying smaller effects than in Lucas (2000) and closer to those in Ireland (2009). More recently, Benati and Nicolini (2021) use data for the United States and several other developed countries and report various estimates depending on the specification used for the money demand function.

In this regard, two money demand specifications have dominated the welfare cost of inflation literature — the semi-log, adapted from Cagan (1956), and the log-log, inspired by Meltzer (1963), with Benati and Nicolini (2021) also using the Selden-Latané specification. Although Benati *et al.* (2021) derive these specifications with a generalized Baumol-Tobin model and Belongia and Ireland (2019) within Sidrauski's (1967) framework, in these specifications the demand for money depends only on the interest rate and the calculation of the welfare cost of inflation requires that one simply integrate under a constant elasticity or semi-elasticity money demand curve as a function of the nominal interest rate.

Recently, Serletis and Xu (2021) develop a new approach to measuring the welfare cost of inflation, within the Bailey (1956) consumer surplus framework. Instead of assuming money demand specifications such as the log-log, semi-log, and Selden-Latané forms, they take a microeconomic- and aggregation theoretic approach to the demand for money paying explicit attention to the demand interactions among consumption goods, leisure, and money, as suggested by Abbott and Ashenfelter (1976) and Barnett (1979). They estimate money demand functions in a systems context based on the Normalized Quadratic (NQ) flexible functional form, developed by Diewert and Wales (1988), and evaluate the cost of inflation conditional on the price of consumption goods, the wage rate, and the user costs of monetary assets. They also use the Divisia monetary aggregates, as suggested by Lucas (2000) and also used by Dai and Serletis (2019), and report time-varying welfare costs of inflation which trend upward over time.

In this paper we provide time-varying estimates of the welfare cost of inflation by extending the Serletis and Xu (2021) methodology. Instead of assuming that consumer preferences are fixed, we follow Xu and Serletis (2022) and assume Markov regime switching, allowing for complicated nonlinear dynamics and sudden changes in the parameters of the aggregator function and the money demand function. In particular, we model the NQ expenditure function as a function of an unobserved regime-shift variable, governed by a first-order two-state Markov process, and pay explicit attention to the theoretical regularity conditions of positivity, monotonicity, and curvature. As Barnett (2002, p. 199) put it, without satisfaction of all three theoretical regularity conditions " ... the second-order conditions for optimizing behavior fail, and duality theory fails. The resulting first-order conditions, demand functions, and supply functions become invalid."

As in Dai and Serletis (2019) and Serletis and Xu (2021), we are motivated by Heckman and Serletis (2014, p. 1), who argue that "the Federal Reserve Board and many other central banks around the world continue officially to produce and supply low quality monetary statistics, inconsistent with the relevant aggregation and index-number theory," and use the Center for Financial Stability (CFS) Divisia monetary aggregates. We use the monthly data, from 1967:1 to 2021:9 (a period that also includes the coronavirus recession), make comparisons among the narrow and broad Divisia money measures, and generate inference in terms of welfare cost of inflation estimates that make full use of the relevant economic theory and econometrics.

We use monthly data, as in Serletis and Xu (2021), and Markov regime switching, as in Dai and Serletis (2019) and Xu and Serletis (2022). We find that the demand interactions between goods, leisure, and money are of significant quantitative importance and that the regimes of high and low welfare cost of inflation vary over the choice of the Divisia monetary aggregate. The welfare cost of a 10% inflation rate with our preferred Divisia M4 monetary aggregate is 1.34% of GDP in the high welfare cost of inflation regime, consistent with Serletis and Xu (2021) which reports a welfare cost of 1.40% of GDP using constant parameters. Our estimates suggest that raising the inflation target in the United States, as it was suggested during the global financial crisis and also recently in the aftermath of the coronavirus pandemic, would impose significant costs. These costs should be taken seriously by those who think that raising inflation targets is a good way to deal with issues that arise by missing existing inflation targets.

The paper proceeds as follows. Section 2 discusses the method for calculating the welfare cost of inflation in the tradition of Bailey (1956). It also discusses related neoclassical demand theory, applied consumption analysis, and econometric issues. In Section 3, we present the Markov regime switching NQ money demand function. Section 4 discusses the data and Section 5 presents the empirical results. The final section concludes.

2 Background

One method for calculating the welfare cost of inflation is the Bailey (1956) approach, associated with consumer surplus analysis in the literature of public finance and applied microeconomics. It has been pursued by Lucas (2000), Cysne (2003), Ireland (2009), Mogliani and Urga (2018), Miller *et al.* (2019), Dai and Serletis (2019), Serletis and Xu (2021), and Benati and Nicolini (2021). In this approach, the welfare cost of inflation is defined as the change in the area under the inverse money demand curve corresponding to the change in the holdings of real money balances — the consumer surplus that can be gained by reducing the nominal interest rate, R, from a positive level to zero. In particular, if $m(R)$ is the money demand function (with m being the ratio of nominal money balances to nominal income) and $\Psi(x)$ its inverse, then the welfare cost of inflation, $\omega(R)$, expressed as a fraction of income, is

$$\omega(R) = \int_{m(R)}^{m(0)} \Psi(x)dx = \int_0^R m(x)dx - Rm(R). \tag{1}$$

In this approach, the first step in the calculation of the welfare cost of inflation is the estimation of a money demand function. In this regard, Lucas (2000) suggests two competing specifications. One is linear in the (natural) logarithms of m (the ratio of nominal money balances to nominal income, M/Y) and R

$$\ln m = \ln A - \eta \ln R \tag{2}$$

where $A > 0$ is a constant and $\eta > 0$ is the interest elasticity of money demand. This specification was inspired by Meltzer (1963) and is known as the log-log (or double log) specification. The other specification was adapted from Cagan (1956) and is known as the semi-log specification

$$\ln m = \ln B - \xi R \tag{3}$$

where $B > 0$ is a constant and $\xi > 0$ is the interest semi-elasticity of money demand. The key difference between (2) and (3) is the coefficient of the interest rate term. In equation (2), η measures the absolute value of the interest elasticity of money demand, while ξ in equation (3) measures the absolute value of the interest semi-elasticity of money demand.

Serletis and Xu (2021) provide a major advance to the literature on measuring the welfare cost of inflation by using a more sophisticated money demand specification in the context of neoclassical demand theory and applied consumption analysis. They integrate the demand for money with the demands for consumption goods and leisure and estimate a flexible money demand specification in a systems context, paying explicit attention to the demand interactions among consumption goods, leisure, and money. They assume identical agents with preferences given by

$$u = u(c, \ell, m, a) \tag{4}$$

where c is real consumption, ℓ is leisure time, m is a Divisia monetary aggregate, and a denotes another Divisia monetary aggregate which includes the monetary assets not included in m. The representative agent's optimization problem is written as

$$\max_{\boldsymbol{x}} u(\boldsymbol{x}) \quad \text{subject to} \quad \boldsymbol{p}'\boldsymbol{x} \leq y \tag{5}$$

where $\boldsymbol{x} = (c, \ell, m, a)$, $\boldsymbol{p}$ is the corresponding vector of prices, and y denotes the total expenditure on $\boldsymbol{x}$. The solution of the first-order conditions is the Marshallian demand functions $\boldsymbol{x} = \boldsymbol{x}(\boldsymbol{p}, y)$ and the indirect utility function $h(\boldsymbol{p}, y)$. Since the Marshallian demand functions are homogenous of degree zero in $\boldsymbol{p}$ and y, the demand system can be written in budget share form as

$$\boldsymbol{w} = \boldsymbol{w}(\boldsymbol{v})$$

where $\boldsymbol{w} = (w_1, ..., w_n)'$, with $w_j = p_j x_j(\boldsymbol{p}, y)/y$, is the expenditure share of good j, and $\boldsymbol{v}$ denotes the income normalized price vector, $\boldsymbol{p}/y$, with the jth element $v_j = p_j/y$.

As discussed in Barnett and Serletis (2008), there are many alternatives for the functional form of the indirect utility function, $h(\boldsymbol{v})$. Serletis and Xu (2021) derive the demand system by approximating the corresponding expenditure function, using the NQ function, introduced by Diewert and Wales (1988). In particular, the NQ expenditure function is

$$C(\boldsymbol{p}, u) = \boldsymbol{\theta}'\boldsymbol{p} + \left(\boldsymbol{b}'\boldsymbol{p} + \frac{1}{2}\frac{\boldsymbol{p}'\boldsymbol{B}\boldsymbol{p}}{\boldsymbol{\alpha}'\boldsymbol{p}}\right) u \tag{6}$$

where $\boldsymbol{\theta} = (\theta_1, \ldots, \theta_n)'$, $\boldsymbol{b} = (b_1, \ldots, b_n)'$, and the elements of the $n \times n$ matrix $\boldsymbol{B} \equiv [\beta_{ij}]$ are the unknown parameters to be estimated. They follow Diewert and Wales (1988) and impose the following restrictions

$$\sum_{i=1}^{n} \alpha_i p_i^* = 1, \quad \alpha_i \geq 0 \quad \forall i \tag{7}$$

$$\sum_{i=1}^{n} \theta_i p_i^* = 0 \tag{8}$$

and

$$\sum_{j=1}^{n} \beta_{ij} p_j^* = 0 \quad \forall i \quad \text{and} \quad \beta_{ij} = \beta_{ji}, \quad \forall i, j \tag{9}$$

where $\boldsymbol{p}^* \gg \mathbf{0}_n$ is a reference (or base-period) vector of normalized prices, determined in such a way that $\boldsymbol{p}^* = \mathbf{1}_n$. The non-negative vector $\boldsymbol{\alpha} = (\alpha_1, \ldots, \alpha_n)'$ is predetermined as a vector of ones ($\boldsymbol{\alpha} = \mathbf{1}_n$) — see Diewert and Wales (1988) for more details.

Serletis and Xu (2021) estimate the demand system corresponding to (6) by considering a stochastic version, assuming that the observed share in the jth equation deviates from the true share by an additive disturbance term ϵ_j. Their stochastic specification is written in matrix notation as

$$\boldsymbol{w}_t = \boldsymbol{w}\left(\boldsymbol{v}_t; \boldsymbol{\theta}\right) + \boldsymbol{\varepsilon}_t \tag{10}$$

where $\boldsymbol{\varepsilon}_t = (\epsilon_{1t}, ..., \epsilon_{nt})'$ is a vector of classical disturbance terms and $\boldsymbol{\theta}$ is the parameter vector to be estimated. It is also assumed that the resulting disturbance vector $\boldsymbol{\varepsilon}$ is a classical disturbance term, $\boldsymbol{\varepsilon}_t \sim N\left(\mathbf{0}, \boldsymbol{H}\right)$, where $\mathbf{0}$ is a null vector and $\boldsymbol{H}$ is the $n \times n$ symmetric positive definite error covariance matrix. Also, since the shares in (10) satisfy the adding up property (that is, they sum to one) and the errors also satisfy adding up (they sum to zero), the error covariance matrix $\boldsymbol{H}$ is singular. This introduces a technical problem when the demand system is estimated, since either generalized least squares or maximum likelihood (ML) needs to invert the covariance matrix, $\boldsymbol{H}$. They follow Barten (1969) who shows that invariant ML estimates can be obtained by arbitrarily dropping any good (or, equivalently, equation) in the system. Also, if the concavity of the NQ expenditure function is not satisfied, in the sense that the estimated $\boldsymbol{B}$ matrix is not negative semidefinite, they follow Diewert and Wales (1988), and impose global concavity by setting $\boldsymbol{B} = -\boldsymbol{K}\boldsymbol{K}'$, where $\boldsymbol{K} = [k_{ij}]$ is a lower triangular matrix.

Serletis and Xu (2021) obtain the demand for Divisia money, m, from the the third equation of the NQ demand system. Their NQ money demand function, unlike the money demand specifications (2) and (3), captures the demand interactions between the four goods, c, ℓ, m, and a. It depends on the total expenditure on consumption goods, leisure, and the services of m and a, y_t, and on the expenditure-normalized price of c, ℓ, m, and a. Moreover, it satisfies all three theoretical regularity conditions.

3 Markov Regime Switching Money Demand

In this paper, we build on Serletis and Xu (2021) and consider the utility function (4) with c being real consumption, ℓ leisure time, m the real Divisia M1 aggregate based on the most recent definition used at the Center for Financial Stability (containing the first six monetary assets listed in Table 1 — currency, x_1, traveler's checks, x_2, demand deposits, x_3, other liquid deposits, x_4, other checkable deposits at commercial banks, x_5, and other checkable deposits at thrift institutions, x_6), and a being another real Divisia monetary aggregate which includes the non-M1 monetary assets listed in Table 1 — assets x_7 to x_{21}. However, we assume that consumer preferences change in response to shocks that hit the economy and relax the assumption of constant parameters in the aggregator function (and thus the demand system), by taking the Markov-switching approach, associated with Hamilton (1989).

We model (10) as a function of an unobserved regime-shift variable, z_t, that follows a first-order Markov process governed by the transition matrix

$$\mathbf{\Pi} = \begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix} \tag{11}$$

where $p_{ji} = P[z_t = j \,|z_{t-1} = i]$, $i, j = 1, 2$, and $p_{ji} = 1 - \sum_{k \neq j} p_{ki}$ is the probability of regime j in period t given that the system was in regime i in period $t-1$. In this paper, we assume two regimes; that is, two exogenous and unobservable states of the economy implied by consumer preferences. The states could be generic as, for example, in Hamilton (1989), expansion and recession. However, given that the states are not observable prior to estimation, they can also be other types of high and low states of the economy. In Section 5, we will show that the states are related to the shape of the money demand function.

Thus, the stochastic Markov regime switching demand system (10) is written as

$$\boldsymbol{w}_{t,z_t} = \boldsymbol{w}\left(\boldsymbol{v}_t; \boldsymbol{\theta}_{z_t}\right) + \boldsymbol{\varepsilon}_{t,z_t} \tag{12}$$

which under the NQ functional form is written as

$$\boldsymbol{w}_{t,z_t}\left(\boldsymbol{v}\right) = \widehat{\boldsymbol{v}}\boldsymbol{\theta}_{t,z_t} + \widehat{\boldsymbol{v}}\frac{\boldsymbol{b}_{t,z_t} + \left(\boldsymbol{\alpha}'_{t,z_t}\boldsymbol{v}\right)^{-1}\boldsymbol{B}_{t,z_t}\boldsymbol{v} - \frac{1}{2}(\boldsymbol{\alpha}'_{t,z_t}\boldsymbol{v})^{-2}\boldsymbol{v}'\boldsymbol{B}_{t,z_t}\boldsymbol{v}\boldsymbol{\alpha}_{t,z_t}}{\boldsymbol{b}'_{t,z_t}\boldsymbol{v} + \frac{1}{2}\left(\boldsymbol{\alpha}'_{t,z_t}\boldsymbol{v}\right)^{-1}\boldsymbol{v}'\boldsymbol{B}_{t,z_t}\boldsymbol{v}} \times (\mathbf{1} - \boldsymbol{\alpha}_{t,z_t}\boldsymbol{v}) + \boldsymbol{\varepsilon}_{t,z_t}$$

where $\widehat{\boldsymbol{v}}$ is the $n \times n$ diagonal matrix with normalized prices on the main diagonal, and $\mathbf{1}_n = [1, \ldots, 1]'$ is a vector of ones.

4 The Data

We use monthly data for the United States from 1967:1 to 2021:9. As in Serletis and Xu (2021), we use the personal consumption expenditure (PCE) series and its price index (the PCE deflator), p_1, from the Federal Reserve Bank of St. Louis FRED database. We convert the personal consumption expenditure series into real per capita terms by dividing by the product of the PCE deflator and population. We use average hourly earnings and average weekly hours of production and non-supervisory employees from FRED, and calculate leisure time, following Hjertstrand *et al.* (2016), as 98 hours per week minus average hours worked per week.

For each of the 21 monetary assets listed in Table 1, we use quantity and real user cost data from the Center for Financial Stability, over the period from 1967:1 to 2021:9. The real user cost of asset j during period t, derived in Barnett (1978, 1980), is given by

$$\pi_{jt} = \frac{R_t - r_{jt}}{1 + R_t} \tag{13}$$

where r_{jt} is the yield on the jth asset and R_t is the yield on an alternative asset (called benchmark asset). See Barnett *et al.* (2013) for details regarding the monetary data. We convert all the monetary quantity series to real per capita terms by dividing by the product of the CPI and total population.

Finally, we use the Divisia index to calculate each of m and a, and their prices (user costs), p_3 and p_4, respectively, as in Serletis and Xu (2021). In particular, the Divisia M1 monetary aggregate and its user cost, p_3, are calculated using quantity and user cost data for assets x_1 to x_6 in Table 1. Similarly, a and its user cost, p_4, are calculated using quantity and user cost data for assets x_7 to x_{21}.

5 Empirical Evidence

We estimate the Markov-switching NQ model with the curvature conditions imposed. All three theoretical regularity conditions — positivity, monotonicity, and concavity — are satisfied at every point in the data set and we report the parameter estimates (together with their standard errors) in columns 1 and 2 of Appendix Table A1, under Divisia M1. In Appendix Figure A1, we plot the smoothed probabilities of each regime, $p(z_t = i|\Omega)$, for $i = 1, 2$, where Ω is the full sample information. We find that regime 1 covers the pre-1975 period, the late 1990s, and the period after the global financial crisis, suggesting that consumer preferences over consumption, leisure, and liquid assets vary as the economy switches between economic expansions and contractions.

5.1 Elasticities

To interpret the parameter estimates reported in column 1 of Appendix Table A1, we turn to an examination of the income elasticities, own- and cross-price elasticities, and the Allen and Morishima elasticities of substitution. All elasticities reported in this paper are mean elasticities, calculated using the formulas used by Serletis and Shahmoradi (2005), and acquired using numerical differentiation.

The regime-dependent income elasticities are reported in panel A of Appendix Table A2, for each of the four goods, c, ℓ, m (Divisia M1), and a (Divisia (M4-M1)). The income elasticities, η_c, η_ℓ, and η_a, are all positive, suggesting that c (consumption), ℓ (leisure), and a (Divisia (M4-M1)) are all normal goods, irrespective of the regime. In panel B of Appendix Table A2 we also report the own- and cross-price elasticities for the four goods. The own-price elasticities, η_{ii}, are all negative, with the absolute values of these elasticities being less than 1, which indicates that the demands for all four goods are inelastic, with leisure being more inelastic ($\hat{\eta}_{\ell\ell} = -0.0287$ with a standard error of 0.0052 in regime 1 and $\hat{\eta}_{\ell\ell} = -0.0302$ with a standard error of 0.0081 in regime 2), followed by Divisia (M4-M1) money ($\hat{\eta}_{aa} = -0.1733$ with a standard error of 0.0757 in regime 1 and $\hat{\eta}_{aa} = -0.0672$ with a standard error of 0.0148 in regime 2) and Divisia M1 money ($\hat{\eta}_{mm} = -0.2556$ with a standard error of 0.0859 in regime 1 and $\hat{\eta}_{mm} = -0.1245$ with a standard error of 0.0268 in regime 2). For the cross-price elasticities, η_{ij}, economic theory does not predict any signs, but we note that most of the off-diagonal terms are negative, indicating that the goods taken as a whole are gross complements.

In addition to the standard Marshallian income and price elasticities, we report estimates of the Allen and Morishima elasticities of substitution in Appendix Tables A3 and A4, respectively. In Appendix Tables A3, we expect the four diagonal terms, representing the Allen own-elasticities of substitution for the four goods to be negative in each of the two regimes, and this expectation is clearly achieved. However, because the Allen elasticity of substitution produces ambiguous results off-diagonal, we use the Morishima elasticity of substitution — the correct measure of substitution [see Blackorby and Russell (1989)] — to investigate the substitutability/complementarity relation between the goods. Based on the Morishima elasticities of substitution reported in Appendix Tables A4, the goods are Morishima substitutes, except for σ^m_{ac} ($\hat{\sigma}^m_{ac} = -0.1379$ with a standard error of 0.0344 in regime 1 and $\hat{\sigma}^m_{ac} = -0.2223$ with a standard error of 0.0377 in regime 2). Moreover, all Morishima elasticities of substitution are less than one.

We show all the time series Morishima elasticities of substitution in Appendix Figures A5-A19. The changing demand system parameters across regimes reflect the changing consumer preferences and cause significant swings in the Morishima elasticities of substitution. Within each regime, the fluctuations in the Morishima elasticities of substitution relate to variations in the prices of consumption and leisure, the monetary assets user costs, and total consumer expenditure, since in our approach the demands for consumption, leisure, and assets depend on all prices and income.

5.2 The NQ Money Demand Function

We obtain the regime-dependent demand for Divisia M1 money by writing the third equation of the NQ demand system, $w_3 = p_3 x_3(\boldsymbol{p}, y)/y$, as

$$x_{3t,z_t} = \frac{y_t}{p_{3t}} \left\{ \theta_{3,z_t} v_{3t} + \frac{b_{3,z_t} + \dfrac{\left(\sum_{j=1}^{n} \beta_{ij,z_t} v_{it}\right)}{\left(\sum_{i=1}^{n} \alpha_{i,z_t} v_{it}\right)} - \dfrac{1}{2} \dfrac{\left(\alpha_{i,z_t} \sum_{k=1}^{n} \sum_{j=1}^{n} \beta_{kj,z_t} v_{kt} v_{jt}\right)}{\left(\sum_{i=1}^{n} \alpha_{i,z_t} v_{it}\right)^2}}{\sum_{i=1}^{n} b_{i,z_t} v_{it} + \dfrac{1}{2} \dfrac{\left(\sum_{i=1}^{n} \sum_{j=1}^{n} \beta_{ij,z_t} v_{it} v_{jt}\right)}{\left(\sum_{i=1}^{n} \alpha_{i,z_t} v_{it}\right)}} \times \left(1 - \sum_{i=1}^{n} \theta_{i,z_t} v_{it}\right) v_{it} \right\} \tag{14}$$

According to (14), the demand for Divisia M1 money captures the demand interactions between the four goods, c, ℓ, m, and a. It depends on the total expenditure on consumption goods, leisure, and the services of m and a, y_t, and on the expenditure-normalized price of c, $\nu_{1t} = p_{1t}/y_t$, wage rate, $\nu_{2t} = p_{2t}/y_t$, user cost of Divisia M1, $\nu_{3t} = p_{3t}/y_t$, and user cost of Divisia (M4-M1), $\nu_{4t} = p_{4t}/y_t$.

To see how well the NQ demand system captures the demand for Divisia M1, in panel A of Figure 1 we plot the real per capita Divisia M1 money demand, x_3, against the nominal interest rate, R. Clearly, there is a negative relation between R (on the vertical axis) and x_3 (on the horizontal axis). In panels B and C of Figure 1 we plot the fitted NQ real per capita Divisia M1 money demand in regimes 1 and 2, respectively. As can be seen, the NQ model provides a very good fit of the monthly US data.

5.3 The Welfare Cost of Inflation

The NQ Divisia M1 money demand function (14) depends on the real user costs of the 21 monetary assets listed in Table 1. As in Serletis and Xu (2021), we follow Kurlat (2019) and model the interest rate spreads in equation (13), $\varsigma_{jt} = R_t - r_{jt}$, as a linear function of the nominal (Center for Financial Stability benchmark) interest rate, R_t, as $\varsigma_{jt} = \phi_0^j + \phi_2^j R_t$ (for $j = 1, ..., 21$), and use ordinary least squares to estimate the ϕ_0^j and ϕ_2^j (for $j = 1, ..., 21$) parameters. Next we use the fitted equation, $\hat{\varsigma}_{jt} = \hat{\phi}_0^j + \hat{\phi}_2^j R_t$, to calculate the Divisia price index for each of Divisia M1 and Divisia (M4-M1) as

$$p_t = \prod_{j=1}^{n} [\pi_{jt}/\pi_{j,t-1}]^{s_j^*}) p_{t-1} = \prod_{j=1}^{n} [\left(\frac{R_t - r_{jt}}{1 + R_t} \right) / \pi_{j,t-1}]^{s_j^*} \tag{15}$$

with $j = 1, ..., 6$ in the case of the Divisia M1 price index, p_3, and $j = 7, ..., 21$ in the case of the Divisia (M4-M1) price index, p_4. In equation (15), $s_{jt}^* = (1/2)(s_{jt} + s_{j,t-1})$, for $j = 1, ..., n$, with $s_{jt} = \pi_{jt} x_{jt} / \sum_{k=1}^{n} \pi_{kt} x_{kt}$ being the expenditure share of asset j, and π_{jt} the real user cost of asset j during period t. See Serletis and Xu (2021) for more details regarding the calculation of the Divisia price index for each of Divisia M1 and Divisia (M4-M1) based on equation (15).

Next, we substitute equation (15) in (14), to get the Divisia M1 money demand function as a function of the nominal interest rate, R, holding all other variables constant, $x_3(R)$. Finally, the welfare cost of inflation, $\omega(R)$, can be calculated as in equation (1), with the integrals in equation (1) being calculated using the *integral* function in Matlab.

To provide a comparison with the estimates available in the literature, we convert the monthly real per capita welfare cost of inflation to nominal terms (multiplying by the CPI) and then in aggregate terms (multiplying by total population). The monthly estimates are then averaged to obtain quarterly figures which are expressed as a fraction of income (dividing by nominal GDP). Finally, the regime-dependent welfare cost of inflation is weighted based on the smoothed regime probabilities, and we present in panel D of Figure 1 the (time-varying) welfare costs of 2%, 3%, 4%, 5%, and 10% inflation rates (for comparison purposes), relative to the benchmark of zero inflation in each of the two regimes.

As can be seen, the welfare cost of inflation is relatively high in regime 1. Moreover, in regime 1 the welfare cost of 10% inflation is significantly higher than the welfare costs of 5%, 4%, 3%, and 2% inflation. The mean values, over the sample period from 1967:1 to 2021:9, are 0.4279% and 0.2309%, 0.1869%, 0.1415%, and 0.0948%, respectively. Also, the welfare cost estimate of 10% inflation is about half of that reported by Lucas (2000), but significantly higher than the estimates reported by Ireland (2009) and Dai and Serletis (2019). In regime 2, the welfare cost of inflation is relatively low, with the mean values being lower than one third of the ones reported for regime 1, 0.1317% and 0.0688%, 0.0553%, 0.0415%, and 0.0276%, respectively.

It is also to be noted that the welfare cost of inflation fluctuates within each regime, with the fluctuations being caused by the time-varying prices of consumption goods and leisure as well as the user costs of monetary assets. Our estimates suggest that the United States economy has been in the high welfare cost of inflation regime since the recovery from the global financial crisis in 2012. In panel A of Table 2 we report the empirical distributions of welfare costs for 2%, 3%, 4%, 5%, and 10% inflation rates. The maximum values are observed in 2021, implying that the welfare cost of inflation surged significantly during the coronavirus pandemic. Thus, the current high inflation rate in excess of 8% has serious consequences in terms of people's welfare which policymakers need to address.

5.4 Broad Divisia Money

In this section we investigate the robustness of our results to broader Divisia monetary aggregates, Divisia M2, Divisia M3, and Divisia M4, as suggested by Lucas (2000). In this regard, Jadidzadeh and Serletis (2019) address the issue of optimal monetary aggregation in the context of a large demand system, encompassing the full range of monetary assets, and support and reinforce Barnett's (2016) assertion that we should use, as a measure of money, the broader Divisia monetary aggregates prepared by the Center for Financial Stability, as opposed to narrower aggregates such as Divisia M1. More recently, Dery and Serletis (2021) also provide evidence that favors the group of broad monetary aggregates.

As in Serletis and Xu (2021), we consider three optimization problems, similar to that in (5), one for each of the Divisia M2, Divisia M3, and Divisia M4 monetary aggregates. In particular, in the Divisia M2 demand system, m is a Divisia aggregate consisting of assets x_1 to x_{16} (in Table 1), and a is the Divisia (M4-M2) aggregate, consisting of the substitute assets x_{17} to x_{21}. In the Divisia M3 demand system, m is a Divisia aggregate consisting of assets x_1 to x_{19}, and a is the Divisia (M4-M3) aggregate, consisting of assets x_{20} and x_{21}. Finally, in the Divisia M4 demand system, we have a three good model — c, ℓ, and m (Divisia M4), since there are no substitute assets as all assets are internalized.

We estimate each of the Divisia M2, Divisia M3, and Divisia M4 NQ demand systems with the curvature conditions imposed, and present the parameter estimates (for each of the two regimes) in Appendix Table A1 under Divisia M2, Divisia M3, and Divisia M4, respectively. The regime probabilities are shown in Appendix Figures A2-A4. Although the identified regimes based on the Divisia M2, Divisia M3, and Divisia M4 monetary aggregates are different, they all show regime-switching during the coronavirus pandemic. We also evaluate the ML parameter estimates by calculating the income elasticities, own- and cross-price elasticities, and the Allen and Morishima elasticities of substitution, which we report in Appendix Tables A5-A7, together (for comparison purposes) with those from the Divisia M1 demand system. As can be seen in Appendix Table A5, the income elasticities are all positive and statistically significant in both regimes and the own-price elasticities are all negative. The Allen own-elasticities of substitution are all negative, as predicted by theory (see Appendix Table A6), and the Morishima elasticities of substitution in Appendix Table A7 indicate Morishima substitutability in both regimes for most pairs of goods. Moreover, the Morishima elasticities of substitution are always less than 1.

In panel A of Figures 2-4, we plot the real per capita Divisia money demand against the nominal interest rate, R, for each of the Divisia M2, Divisia M3, and Divisia M4 aggregates, respectively, in a similar fashion as we did in panel A of Figure 1 with the Divisia M1 aggregate. Again, we observe negative relationships between the nominal interest rate and each of the broad Divisia aggregates. To see how well each of the Divisia M2, Divisia M3, and Divisia M4 NQ demand systems captures the demand for money, we plot the fitted NQ real per capita Divisia money demand in each of the two regimes in panels B and C of Figures 2-4. As can be seen, the NQ model consistently delivers the regime-dependent money demand curves which are summarized from the raw data.

Finally, we present the (time-varying) welfare costs of 2%, 3%, 4%, 5%, and 10% inflation rates in panel D of Figures 2-4, for each of the Divisia M2, Divisia M3, and Divisia M4 aggregates, as we did in panel D of Figure 1 with the Divisia M1 aggregate. We can still classify the two regimes as the high and low welfare cost of inflation regimes. In particular, regime 1 is the high welfare cost of inflation regime for Divisia M1 and Divisia M2, and regime 2 is the high welfare cost of inflation regime for Divisia M3 and Divisia M4. We see that in general, the welfare cost of inflation increases with the level of aggregation, being higher at the M4 level in the high welfare cost of inflation regime. However, the welfare cost of inflation is higher at the M2 level of aggregation in the low welfare cost of inflation regime. We note that all Figures 1-4 indicate that the United States economy is experiencing the high welfare cost of inflation regime. In panels B, C, and D of Table 2 we report the empirical distributions of welfare costs for 2%, 3%, 4%, 5%, and 10% inflation rates, with the Divisia M2, Divisia M3, and Divisia M4 aggregates, respectively, as we did with the Divisia M1 aggregate in panel A of Table 2. Again, the maximum welfare cost of inflation values are observed in 2021.

To summarize the results, in Table 3 we present the mean welfare cost estimates of 2%, 3%, 4%, 5%, and 10% inflation rates, for each of the Divisia M1, Divisia M2, Divisia M3, and

Divisia M4 monetary aggregates. We see that the welfare cost of a 10% inflation rate with the Divisia M4 aggregate is 1.34% of GDP in the high welfare cost of inflation regime. This is consistent with Serletis and Xu (2021), which reports a welfare cost of 1.40% of GDP. It is to be noted that Serletis and Xu (2021) use constant parameter values and might not be capturing the potentially lower welfare cost of inflation in a dynamic economic environment.

According to panel D of Figure 4, the fluctuations in the welfare cost of inflation within each regime are relatively stable. The significant swings are mainly caused by changes in consumer preferences in a dynamic economic environment. Based on the mean values implied by the time series welfare cost of inflation estimates in each regime, we show a range of welfare costs of inflation. For example, we find the welfare cost of inflation ranges from around 0.089% to 1.34% when inflation is allowed to be 10% by monetary policymakers, conditional on the state of the economy.

What explains the high and low welfare costs of inflation? As can be seen in Figures 1-4, the regime-dependent money demand curves have two different slopes for each of the four Divisia monetary aggregates. No matter which Divisia monetary aggregate is used, we always find one money demand curve to be steeper (more inelastic) than the other, generating a smaller welfare cost of inflation, since in that case the increase in inflation does not reduce the holdings of real money balances as much as in the case of a more elastic money demand curve. Our estimates, based on all four Divisia monetary aggregates, suggest that the United States economy is operating at the high welfare cost of inflation regime with a relatively elastic money demand curve. Moreover, the prices of consumption and leisure and the monetary asset user costs during the Covid-19 period are leading to an even higher welfare cost of inflation in the high welfare cost of inflation regime.

As can be seen in Figures A1-A4, the regimes of high and low welfare costs of inflation vary over the choice of monetary aggregate in the utility function and the corresponding demand system. The main reason for this could be that the demand interactions between goods, leisure, and money are likely to be significantly different depending on which assets people consider to be money. In other words, the substitutability/complementarity relationships among consumption, leisure, and money vary depending on what is considered as money, and our estimates of the elasticities of substitution confirm this point. Thus, the regimes that characterize consumer preferences are different across the different Divisia monetary aggregates.

Finally, another important observation is that the pre- and post- Covid periods are important parts of our detected regimes. To investigate the robustness of our results, we re-estimate the model with the turbulent (Covid) tail of the sample removed, and report the results in Appendix Table A2 and Appendix Figures A20-A23. The evidence is in general consistent with our main findings, except that the welfare cost of inflation estimates are lower when the Divisia M3 monetary aggregate is used.

6 Welfare Costs in the Long Run

In the context of the demand systems approach taken in this paper, the term "long run" seems ambiguous and would benefit from a formal discussion. We use a static utility function (4), and in this framework the choice variables fully adjust instantly and take the new equilibrium values following changes in prices or income. However, Markov switching allows consumer preferences to change in response to shocks that hit the economy. For example, changes in monetary policy (such as the introduction of inflation targeting), technological and institutional changes, and large-scale events (such as wars and financial crises) can induce significant shifts in consumer tastes and preferences over consumption, leisure, and monetary assets. Another class of models that allows this to occur is the "time-varying coefficient model," recently used by Miller *et al.* (2019) in their investigation of the welfare cost of inflation in the United States using the log-log and semi-log forms.

With our approach, we can obtain an average (long-run) estimate of the welfare cost of inflation. With a transition matrix as in (11), Hamilton (1994, p. 681-682) shows that

$$\lim_{j\to\infty} \mathbf{\Pi}^j = \boldsymbol{\pi} \begin{pmatrix} 1 \\ 1 \end{pmatrix}' = \begin{pmatrix} \pi_1 \\ \pi_2 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}'$$

where $\boldsymbol{\pi}$ is a vector with elements that sum to unity. Moreover, Hamilton (1994) proves that

$$E(z_{t+n}|z_t,\ z_{t-1}, ...) = \boldsymbol{\pi}.$$

It follows that $\boldsymbol{\pi}$ is the long-run forecast of the probabilities of being in regime i, $i = 1, 2$. In other words, $\boldsymbol{\pi}$ gives the unconditional probability of each of the two regimes. Given the mean values of the welfare cost of inflation in each regime reported in Table 2, we can weigh them by $\boldsymbol{\pi}$ to obtain a single estimate that gives the welfare cost of inflation in the long run, as follows

$$\omega(R)^{\text{Long run}} = \pi_1 \bar{\omega}(R)_{z_t=1} + \pi_2 \bar{\omega}(R)_{z_t=2} \tag{16}$$

where $\bar{\omega}(R)_{z_t=1}$ and $\bar{\omega}(R)_{z_t=2}$ are the mean values of the welfare cost of inflation in the two regimes, as reported in Table 2 for each Divisia monetary aggregate.

The long-run estimates reported in Table 2 (under the 'Long-run' row) show that the welfare cost of inflation is the lowest with the Divisia M1 aggregate. We still get the highest welfare cost with the Divisia M4 monetary aggregate at each of 2%, 3%, 4%, 5%, and 10% inflation rates. In particular, we find that with the Divisia M4 monetary aggregate the long-run welfare cost is 0.7296% when the inflation is 10%.

Even though our estimates of the welfare costs of inflation based on the Divisia M4 monetary aggregate are not directly comparable to those based on conventional approaches and primarily at the M1 level of monetary aggregation, our results provide important information across different definitions of Divisia money. Compared with established results in the literature, our estimate of the long-run welfare cost of inflation with the Divisia M4 monetary aggregate is lower than the estimate reported by Lucas (2000) with the annual data (from 1900 to 1994), the log-log money demand specification, and the simple-sum M1 monetary aggregate. It is also an order of magnitude larger than the estimates reported by Ireland (2009) and Dai and Serletis (2019) with the quarterly data, the semi-log money demand specification, and the simple-sum M1 and Divisia M1 monetary aggregate, respectively.

7 Comparison with Other Studies

It is difficult to provide a comparison between our results and those reported in other studies. As we mentioned in the introduction, the major contributions in this area have employed lower frequency data, either the log-log or semi-log money demand specifications, and the simple-sum M1 monetary aggregate. For example, Lucas (2000) uses annual data over the period from 1900 to 1994, the log-log money demand specification, and the simple-sum M1 monetary aggregate. Also, assuming an interest elasticity of -0.5 (as in the Baumol-Tobin model), Lucas (2000) reports a welfare cost of inflation of about 1% of real income per year if the annual inflation rate is 10%. Serletis and Yavari (2004) use the same data and money demand function used by Lucas (2000), but use the long-horizon regression approach to estimate the interest elasticity of money demand to be -0.21, and report a welfare cost of inflation which is about half of the value reported by Lucas (2000). On the other hand, Ireland (2009) uses quarterly data over the post-1980 period, the semi-log specification, the simple-sum M1 monetary aggregate, and estimates the welfare cost of inflation to be around 0.23% of real income if the annual inflation rate is 10%.

As can be seen in Table 4, most of the more recent studies are accounting for instabilities in the long-run money demand function in measuring the welfare cost of inflation. For example, Mogliani and Urga (2018) find structural breaks and estimate a substantially lower welfare cost of inflation after 1976. Dai and Serletis (2019) take issue with the assumption of an exogenous structural break in Ireland (2009), and use the Markov switching approach, treating the structural break as endogenous. Miller *et al.* (2019) use the time-varying coefficient model and report time-varying long-run welfare costs of inflation. The present paper follows Serletis and Xu (2019) and Xu and Serletis (2022) and uses neoclassical demand theory to integrate the demand for money with the demands for consumption and leisure. It estimates a flexible money demand function in a systems context using Markov regime switching and broad Divisia monetary aggregates. We estimate the welfare cost of inflation in a manner making full use of all relevant microeconomic theory, including aggregation theory to aggregate over component monetary assets, nested within a consumer demand system of equations derived to be integrable to a utility function.

Our results are also important, implying that the welfare cost of inflation is high. They support the recent unprecedented contractionary monetary policy (with both increases in policy rates and quantitative tightening) by central banks in advanced economies to fight the post Covid-19 persistent inflation and anchor inflation expectations.

8 Conclusion

This paper provides an advance to the literature on measuring the welfare costs of inflation that results from the use of more sophisticated demand for money function specifications and Divisia monetary aggregates. It uses an econometric framework that flexibly allows for changes in the coefficients of the money demand function and address the 'Barnett Critique,' first defined by Chrystal and MacDonald (1994) and more recently explored by Belongia and Ireland (2014). By making use of the relevant economic theory and econometrics, it removes an internal inconsistency between the theory that produces the Divisia monetary aggregates and the money demand specifications being used in the literature on measuring the welfare costs of inflation.

Our results also raise the question of which monetary aggregate to use in measuring the welfare costs of inflation. In this regard, Jadidzadeh and Serletis (2019) address the issue of optimal monetary aggregation in the context of a large demand system, encompassing the full range of monetary assets, and support and reinforce Barnett's (2016) assertion that we should use, as a measure of money, the broader Divisia monetary aggregates prepared by the Center for Financial Stability, as opposed to narrower aggregates such as Divisia M1. More recently, Dery and Serletis (2021) also provide evidence that favors the group of broad Divisia monetary aggregates. We conclude that the welfare costs of inflation are far from trivial when computed using the broad Divisia monetary aggregates.

References

[1] Abbott, M., Ashenfelter, O., 1976. Labor supply, commodity demand, and the allocation of time. *Review of Economic Studies* 43, 339-412.

[2] Bailey, M.J., 1956. The welfare cost of inflationary finance. *Journal of Political Economy* 64, 93-110.

[3] Barnett, W.A., 1978. The user cost of money. *Economics Letters* 1, 145-149.

[4] Barnett, W.A., 1979. The joint allocation of leisure and goods expenditure. *Econometrica* 47, 539-563.

[5] Barnett, W.A., 1980. Economic monetary aggregates: An application of index number and aggregation theory. *Journal of Econometrics* 14, 11-48.

[6] Barnett, W.A., 2002. Tastes and technology: Curvature is not sufficient for regularity. *Journal of Econometrics* 108, 199-202.

[7] Barnett, W.A., 2016. Friedman and Divisia monetary measures. In R.A. Cord and D. Hammond (eds.), *Milton Friedman: Contributions to Economics and Public Policy*, pp. 265-291. Oxford: Oxford University Press.

[8] Barnett, W.A., Serletis, A., 2008. Consumer preferences and demand systems. *Journal of Econometrics* 147, 210-224.

[9] Barnett,W.A., Liu, J., Mattson, R.S., van den Noort, J., 2013. The new CFS Divisia monetary aggregates: Design, construction, and data sources. *Open Economies Review* 24, 101-124.

[10] Barten, A.P., 1969. Maximum likelihood estimation of a complete system of demand equations. *European Economic Review* 1, 7-73.

[11] Belongia, M.T., Ireland, P.N., 2014. The Barnett critique after three decades: A new Keynesian analysis. *Journal of Econometrics* 183, 5-21.

[12] Belongia, M.T., Ireland, P.N., 2019. The demand for Divisia money: Theory and evidence. *Journal of Macroeconomics* 61, 103128.

[13] Benati, L., Nicolini, J.-P., 2021. The welfare costs of inflation. *Federal Reserve Bank of Minneapolis* Working Paper No. 783.

[14] Benati, L., Lucas, R.E. Jr., Nicolini, J.-P., Weber, W., 2021. International evidence on long run money demand. *Journal of Monetary Economics* 117, 43-63.

[15] Blackorby, C., Russell, R.R., 1989. Will the real elasticity of substitution please stand up? *American Economic Review* 79, 882-888.

[16] Cagan, P., 1956. The monetary dynamics of hyperinflation. In *Studies in the Quantity Theory of Money*, edited by Milton Friedman, pp. 25-117. Chicago and London: University of Chicago Press.

[17] Chrystal, K.A., MacDonald, R., 1994. Empirical evidence on the recent behaviour and usefulness of simple-sum and weighted measures of the money stock. Federal Reserve Bank of St. Louis *Review* 76, 73-109.

[18] Cysne, R.P., 2003. Divisia index, inflation, and welfare. *Journal of Money, Credit and Banking* 35, 221-238.

[19] Dai, W., Serletis, A., 2019. On the Markov switching welfare cost of inflation. *Journal of Economic Dynamics and Control* 108, 103748.

[20] Dery C., Serletis, A., 2021. Interest rates, money, and economic activities. *Macroeconomic Dynamics* 25, 1842-1891.

[21] Diewert, W.E., 1973. Functional forms for profit and transformation functions. *Journal of Economic Theory* 6, 284 316.

[22] Diewert, W.E., 1974. Applications of duality theory. In *Frontiers in Quantitative Economics* Vol 2, M. Intriligator and D. Kendrick (eds.). Amsterdam: North-Holland, pp. 106-171.

[23] Diewert, W.E., Wales, T.J., 1988. Normalized quadratic systems of consumer demand functions. *Journal of Business and Economics Statistics* 6, 303-312.

[24] Hamilton, J.D., 1989. A new approach to the economic analysis of nonstationary time series and the business cycle. *Econometrica* 57, 357-384.

[25] Heckman, J.J., Serletis, A., 2014. Introduction to *internally consistent modeling, aggregation, inference, and policy. Journal of Econometrics* 183, 1-4.

[26] Hjertstrand, P., Swofford, J.L., Whitney, G.A., 2016. Mixed integer programming revealed preference tests of utility maximization and weak separability of consumption, leisure and money." *Journal of Money, Credit and Banking* 48, 1547-1561.

[27] Jadidzadeh, A., Serletis, A., 2019. The demand for assets and optimal monetary aggregation. *Journal of Money, Credit and Banking* 51, 929-952.

[28] Ireland, P.N., 2009. On the welfare cost of inflation and the recent behavior of money demand. *American Economic Review* 99, 1040-1052.

[29] Kurlat, P., 2019. Deposit spreads and the welfare cost of inflation. *Journal of Monetary Economics* 106, 78-93.

[30] Latané, H.A., 1960. Income velocity and interest rates: A pragmatic approach. *Review of Economics and Statistics* 42, 445-449.

[31] Lucas, R.E. Jr., 2000. Inflation and welfare. *Econometrica* 68, 247-274.

[32] Lucas, R.E. Jr., Nicolini, J.-P., 2015. On the stability of money demand. *Journal of Monetary Economics* 73, 48-65.

[33] Meltzer, A.H., 1963. The demand for money: The evidence from the time series. *Journal of Political Economy* 71, 219-246.

[34] Miller, S.M., Martins, L.F., and Gupta, R., 2019. A time-varying approach of the US welfare cost of inflation. *Macroeconomic Dynamics* 23, 775-797.

[35] Mogliani, M., Urga, G., 2018. On the instability of long-run money demand and the welfare cost of inflation in the United States. *Journal of Money, Credit and Banking* 50, 1645-1660.

[36] Selden, R.T., 1956. Monetary velocity in the United States. In M. Friedman (Ed.), *Studies in the Quantity Theory of Money*, University of Chicago Press, pp. 405-454.

[37] Serletis, A., Shahmoradi, A., 2005. Semi-nonparametric estimates of the demand for money in the United States. *Macroeconomic Dynamics* 9, 542-559.

[38] Serletis, A., Xu, L., 2021. The welfare cost of inflation. *Journal of Economic Dynamics and Control* 128, 104144.

[39] Serletis, A., Yavari, K., 2004. The welfare cost of inflation in Canada and the United States. *Economics Letters*, 84, 199-204.

[40] Sidrauski, M., 1967. Rational choice and patterns of growth in a monetary economy. *American Economic Review* 57, 534-544.

[41] Xu, L., Serletis, A., 2022. The demand for assets: Evidence from the Markov switching Normalized Quadratic model. *Journal of Money, Credit and Banking* 54, 989-1025.

Table 1. Monetary assets/components

Mnemonic	Asset/component	Monetary subaggregate			
		$M1$	$M2$	$M3$	$M4$
x_1	Currency	✓	✓	✓	✓
x_2	Traveler's check	✓	✓	✓	✓
x_3	Demand deposits	✓	✓	✓	✓
x_4	Other liquidity deposits	✓	✓	✓	✓
x_5	OCDs at commercial banks	✓	✓	✓	✓
x_6	OCDs at thrift institutions	✓	✓	✓	✓
x_7	Saving deposits at commercial banks		✓	✓	✓
x_8	Saving deposits at thrifts institutions		✓	✓	✓
x_9	Retail money-market funds		✓	✓	✓
x_{10}	MMDAs at commercial banks		✓	✓	✓
x_{11}	MMDAs at thrift institutions		✓	✓	✓
x_{12}	Saving deposits at commercial banks excluding MMDAs		✓	✓	✓
x_{13}	Saving deposits at thrift institutions excluding MMDAs		✓	✓	✓
x_{14}	Small-denomination time deposits		✓	✓	✓
x_{15}	Small time deposits at commercial banks		✓	✓	✓
x_{16}	Small time deposits at thrift institutions		✓	✓	✓
x_{17}	Institutional money-market funds			✓	✓
x_{18}	Large time deposits			✓	✓
x_{19}	Repurchase agreements			✓	✓
x_{20}	Commercial paper				✓
x_{21}	T-bills				✓

Figure 1. The Demand for Divisia M1 (in real per capita terms)

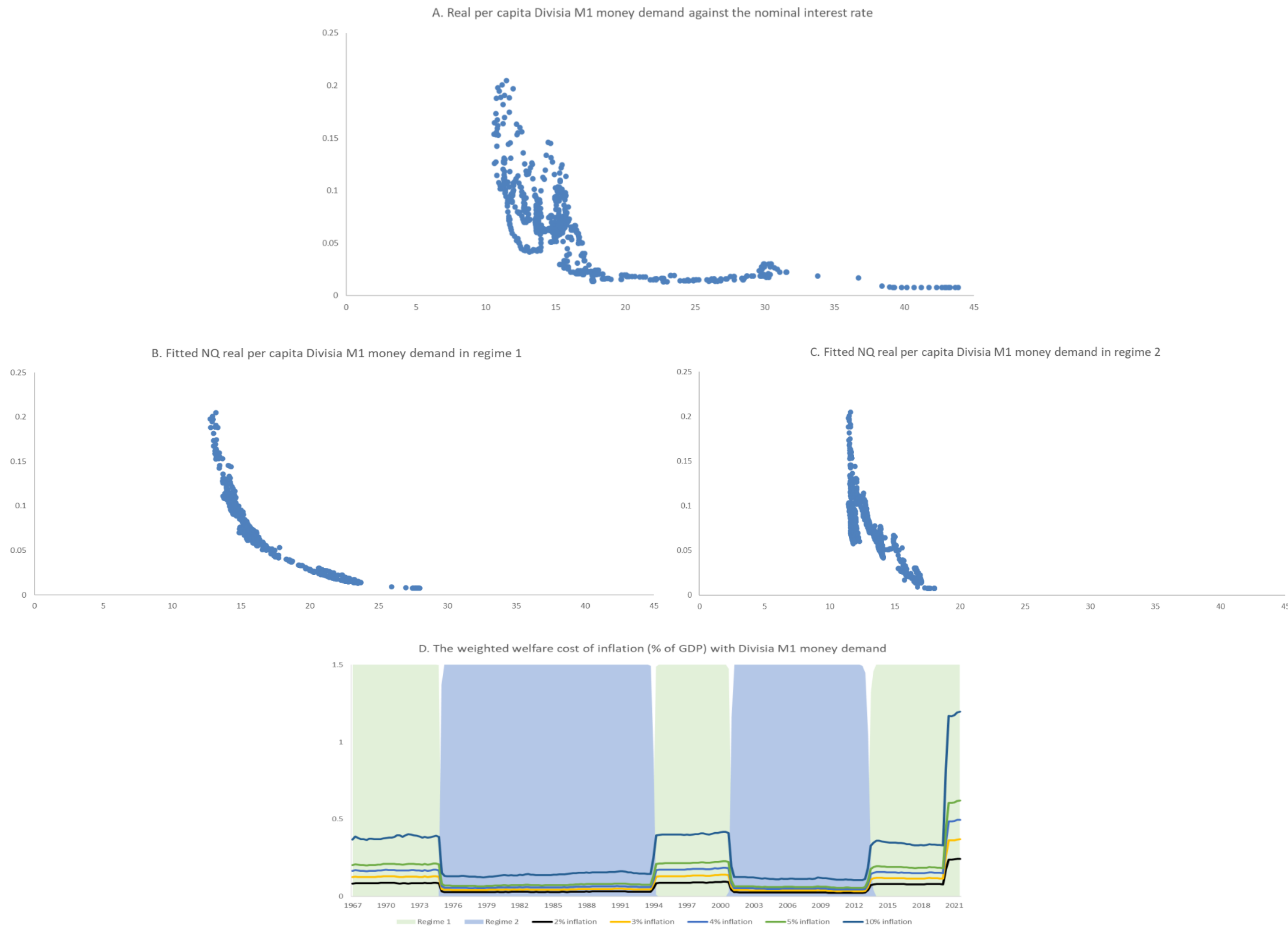


Figure 2. The Demand for Divisia M2 (in real per capita terms)

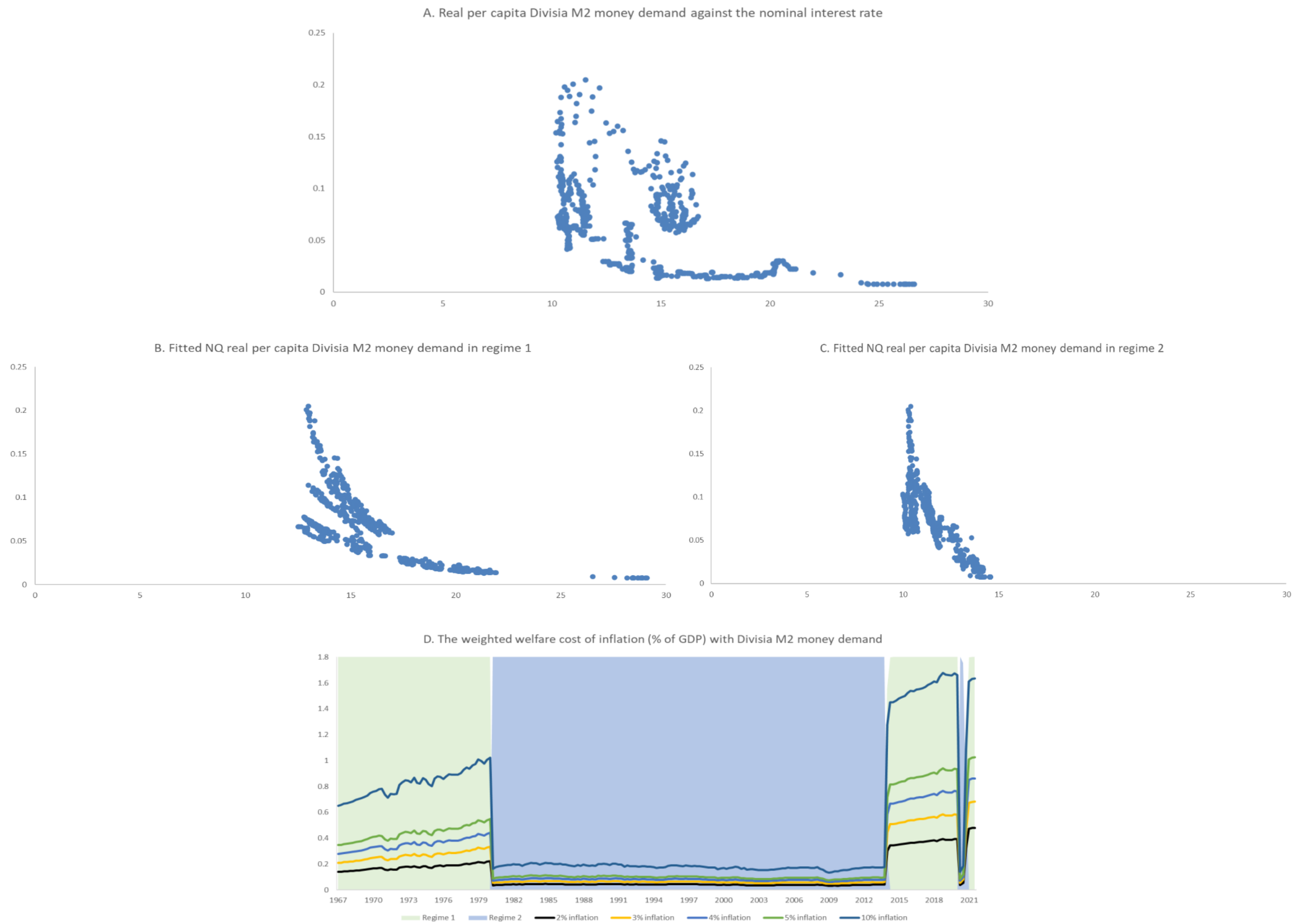


Figure 3. The Demand for Divisia M3 (in real per capita terms)

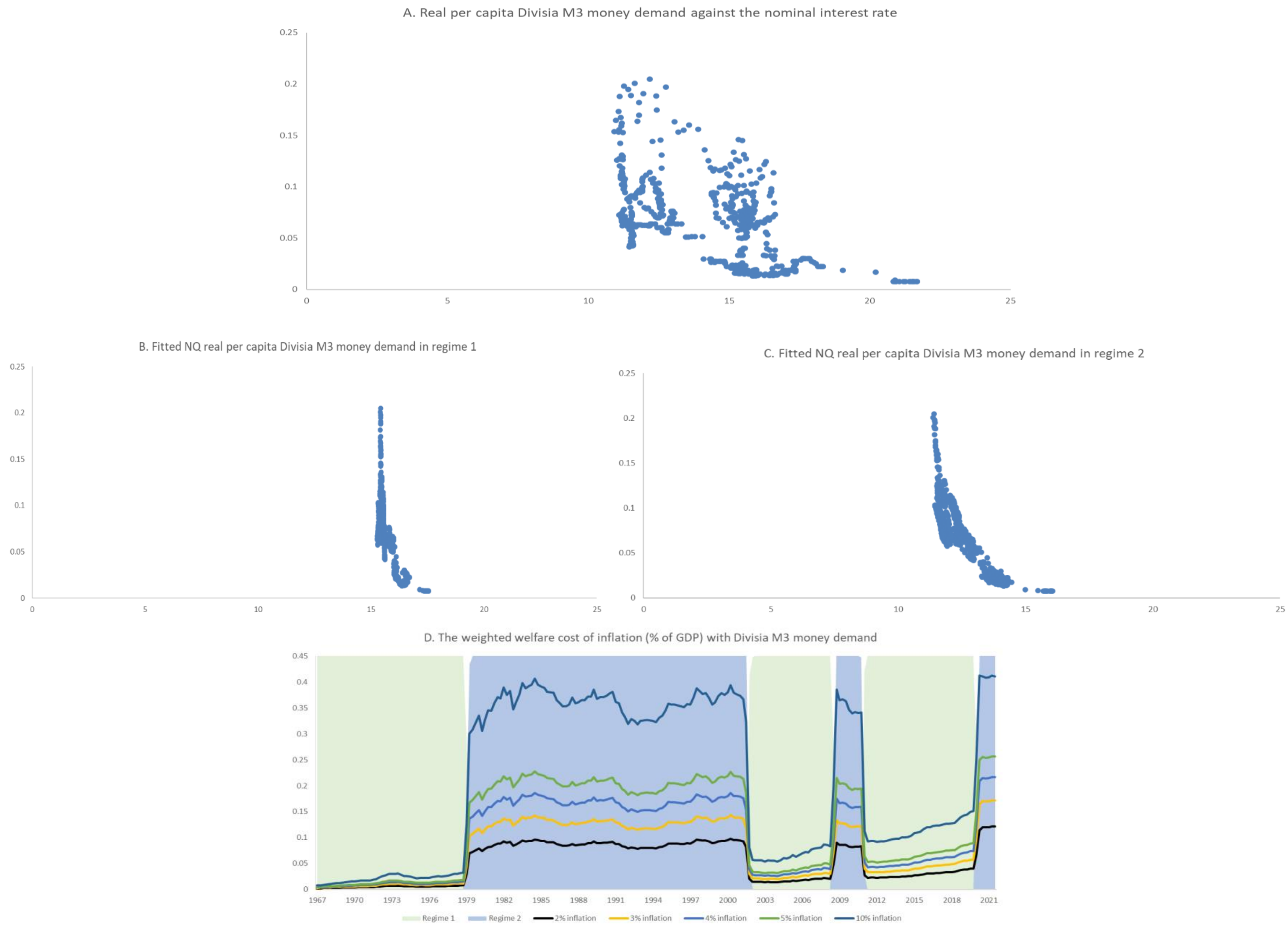


Figure 4. The Demand for Divisia M4 (in real per capita terms)

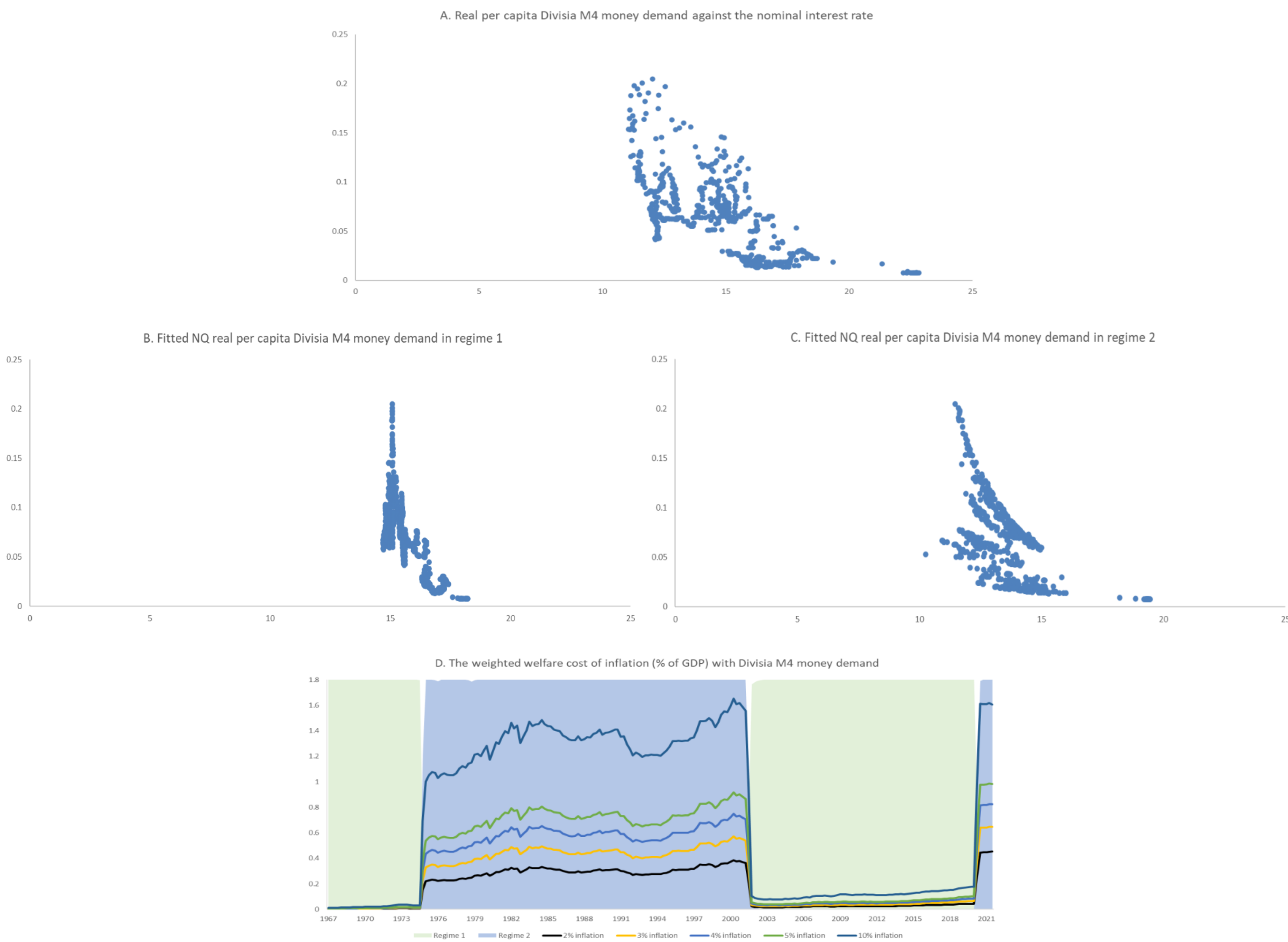


Table 2. Summary statistics of welfare costs, 1967:q1-2021:q3

Inflation rate	Mean	Median	Maximum	Minimum	Standard deviation	Jarque-Bera
			A. Divisia M1			
2% in regime 1	0.0948	0.0849	0.2433 (2021:q3)	0.0778 (2017:q3)	0.0378	625.8440 (0.0000)
3% in regime 1	0.1415	0.1272	0.3697 (2021:q3)	0.1147 (2017:q3)	0.0579	625.7735 (0.0000)
4% in regime 1	0.1869	0.1684	0.4958 (2021:q3)	0.1499 (2017:q3)	0.0783	625.2918 (0.0000)
5% in regime 1	0.2309	0.2089	0.6201 (2021:q3)	0.1835 (2020:q1)	0.0986	624.5965 (0.0000)
10% in regime 1	0.4279	0.3862	1.1976 (2021:q3)	0.3299 (2020:q1)	0.1946	620.4557 (0.0000)
2% in regime 2	0.0276	0.0276	0.0329 (1990:q4)	0.0222 (2012:q3)	0.0028	3.5131 (0.1726)
3% in regime 2	0.0415	0.0414	0.0498 (1990:q4)	0.0333 (2012:q3)	0.0044	4.1040 (0.1285)
4% in regime 2	0.0553	0.0551	0.0666 (1990:q4)	0.0442 (2012:q3)	0.0060	4.6848 (0.0961)
5% in regime 2	0.0688	0.0686	0.0832 (1990:q4)	0.0549 (2012:q3)	0.0077	5.2136 (0.0738)
10% in regime 2	0.1317	0.1309	1.1616 (1991:q2)	0.1043 (2012:q3)	0.0161	6.9624 (0.0308)

Note: Numbers in parentheses indicates the p-value of the Jarque-Bera test.

Table 2. Summary statistics of welfare costs, 1967:q1-2021:q3 (cont.)

Inflation rate	Mean	Median	Maximum	Minimum	Standard deviation	Jarque-Bera
			B. Divisia M2			
2% in regime 1	0.2419	0.1898	0.4796 (2021:q3)	0.1386 (1967:q1)	0.0995	11.4733 (0.0032)
3% in regime 1	0.3604	0.2855	0.6819 (2021:q3)	0.2090 (1967:q1)	0.1437	11.5597 (0.0031)
4% in regime 1	0.4749	0.3798	0.8626 (2021:q3)	0.2784 (1967:q1)	0.1842	11.6409 (0.0030)
5% in regime 1	0.5849	0.4717	1.0246 (2021:q3)	0.3460 (1967:q1)	0.2214	11.6791 (0.0029)
10% in regime 1	1.0664	0.8899	1.6786 (2018:q3)	0.6509 (1967:q1)	0.3676	11.3542 (0.0000)
2% in regime 2	0.0404	0.0407	0.0462 (1984:q4)	0.0316 (2009:q1)	0.0034	7.1666 (0.0278)
3% in regime 2	0.0601	0.0603	0.0691 (1984:q4)	0.0468 (2009:q1)	0.0051	6.4163 (0.0404)
4% in regime 2	0.0792	0.0794	0.0914 (1984:q4)	0.0613 (2009:q1)	0.0068	5.5992 (0.0608)
5% in regime 2	0.0974	0.0977	0.1130 (1984:q4)	0.0752 (2009:q1)	0.0085	4.8105 (0.0902)
10% in regime 2	0.1779	0.1766	0.2102 (1984:q4)	0.1349 (2009:q1)	0.0167	2.1691 (0.3381)

Note: Numbers in parentheses indicates the p-value of the Jarque-Bera test.

Table 2. Summary statistics of welfare costs, 1967:q1-2021:q3 (cont.)

Inflation rate	Mean	Median	Maximum	Minimum	Standard deviation	Jarque-Bera
			C. Divisia M3			
2% in regime 1	0.0157	0.0143	0.0400 (2019:q4)	0.0022 (1967:q1)	0.0110	8.9292 (0.0115)
3% in regime 1	0.0229	0.0209	0.0578 (2019:q4)	0.0032 (1967:q1)	0.0160	8.9357 (0.0031)
4% in regime 1	0.0297	0.0271	0.0743 (2019:q4)	0.0041 (1967:q1)	0.1842	8.9635 (0.0113)
5% in regime 1	0.0360	0.0329	0.0895 (2019:q4)	0.0049 (1967:q1)	0.0248	9.0049 (0.0111)
10% in regime 1	0.0618	0.0562	0.1509 (2019:q4)	0.0081 (1967:q1)	0.0424	9.2825 (0.0097)
2% in regime 2	0.0894	0.0884	0.1213 (2021:q3)	0.0723 (1979:q3)	0.0093	142.0612 (0.0000)
3% in regime 2	0.1310	0.1300	0.1716 (2021:q3)	0.1071 (1979:q3)	0.0124	84.5920 (0.0000)
4% in regime 2	0.1703	0.1689	0.2164 (2021:q2)	0.1404 (1979:q3)	0.0150	45.6166 (0.0000)
5% in regime 2	0.2073	0.2056	0.2568 (2021:q2)	0.1722 (1979:q3)	0.0171	21.5683 (0.0000)
10% in regime 2	0.3630	0.3651	0.4126 (2020:q2)	0.3054 (1980:q3)	0.0250	1.2596 (0.5327)

Note: Numbers in parentheses indicates the p-value of the Jarque-Bera test.

Table 2. Summary statistics of welfare costs, 1967:q1-2021:q3 (cont.)

Inflation rate	Mean	Median	Maximum	Minimum	Standard deviation	Jarque-Bera
			D. Divisia M4			
2% in regime 1	0.0197	0.0227	0.0441 (2020:q1)	0.0020 (1967:q1)	0.0120	3.3943 (0.1832)
3% in regime 1	0.0293	0.0344	0.0644 (2020:q1)	0.0030 (1967:q1)	0.0176	3.6894 (0.1581)
4% in regime 1	0.0386	0.0455	0.0084 (2020:q1)	0.0041 (1967:q1)	0.0228	4.0378 (0.1328)
5% in regime 1	0.0476	0.0561	0.1016 (2020:q1)	0.0052 (1967:q1)	0.0278	4.4136 (0.1101)
10% in regime 1	0.0884	0.1046	0.1780 (2020:q1)	0.0111 (1967:q1)	0.0493	6.2470 (0.0440)
2% in regime 2	0.3054	0.3042	0.4532 (2021:q3)	0.2189 (1975:q1)	0.0468	28.0881 (0.0000)
3% in regime 2	0.4538	0.4540	0.6375 (2021:q3)	0.3279 (1975:q1)	0.0656	14.1532 (0.0008)
4% in regime 2	0.5973	0.5992	0.8242 (2021:q2)	0.4345 (1975:q1)	0.0819	6.5002 (0.0388)
5% in regime 2	0.7353	0.7360	0.9860 (2021:q2)	0.5380 (1975:q1)	0.0964	2.5559 (0.2786)
10% in regime 2	1.3418	1.3503	1.6517 (2000:q2)	1.0023 (1975:q1)	0.1505	1.0390 (0.5948)

Note: Numbers in parentheses indicates the p-value of the Jarque-Bera test.

Table 3. Regime-dependent welfare costs of inflation

Regime	Inflation rate 2%	3%	4%	5%	10%
			Divisia M1		
Regime 1	0.0948	0.1415	0.1869	0.2309	0.4279
Regime 2	0.0276	0.0415	0.0553	0.0688	0.1317
Long-run	0.0638	0.0954	0.1263	0.1562	0.2914
			Divisia M2		
Regime 1	0.2419	0.3604	0.4749	0.5849	1.0664
Regime 2	0.0404	0.0601	0.0792	0.0974	0.1779
Long-run	0.1404	0.2091	0.2756	0.3393	0.6188
			Divisia M3		
Regime 1	0.0157	0.0229	0.0297	0.0360	0.0618
Regime 2	0.0894	0.1310	0.1703	0.2073	0.3630
Long-run	0.0516	0.0756	0.0982	0.1195	0.2086
			Divisia M4		
Regime 1	0.0197	0.0293	0.0386	0.0476	0.0884
Regime 2	0.3054	0.4538	0.5973	0.7353	1.3418
Long-run	0.1658	0.2464	0.3244	0.3994	0.7296

Note: Numbers are mean values (in %).

Table 4. A Comparison with other studies

Study	Demand function	Monetary aggregate	Estimation method	Data	Welfare cost
Lucas (2000)	log-log	Sum M1	OLS	1900-1994, annual	1%
Serletis and Yavari (2004)	log-log	Sum M1	Long horizon regressions	1948-2001, annual	0.45%
Ireland (2009)	semi-log	Sum M1	Dynamic OLS	1980-2006, quarterly	0.23%
Mogliani and Urga (2018)	log-log	Sum M1	Cointegrating regressions	1900-1944, annual	0.4%-0.6%
				1945-1975, annual	1.2%-1.5%
				1976-2013, annual	0.1%-0.3%
Dai and Serletis (2019)	semi-log	Divisia M1	Markov switching	1967-2013, quarterly	
				regime 1	0.3%
				regime 2	0.16%
Miller *et al.* (2019)	log-log	Sum M1	Time-varying cointegration	1959-2010, quarterly	0.27%
Serletis and Xu (2021)	NQ	Divisia M4	Maximum likelihood (ML)	1967-2019, quarterly	1.4%
This study	NQ	Divisia M4	ML with Markov switching	1967-2021, monthly	
				regime 1	0.08%
				regime 2	1.34%
				long run	0.73%

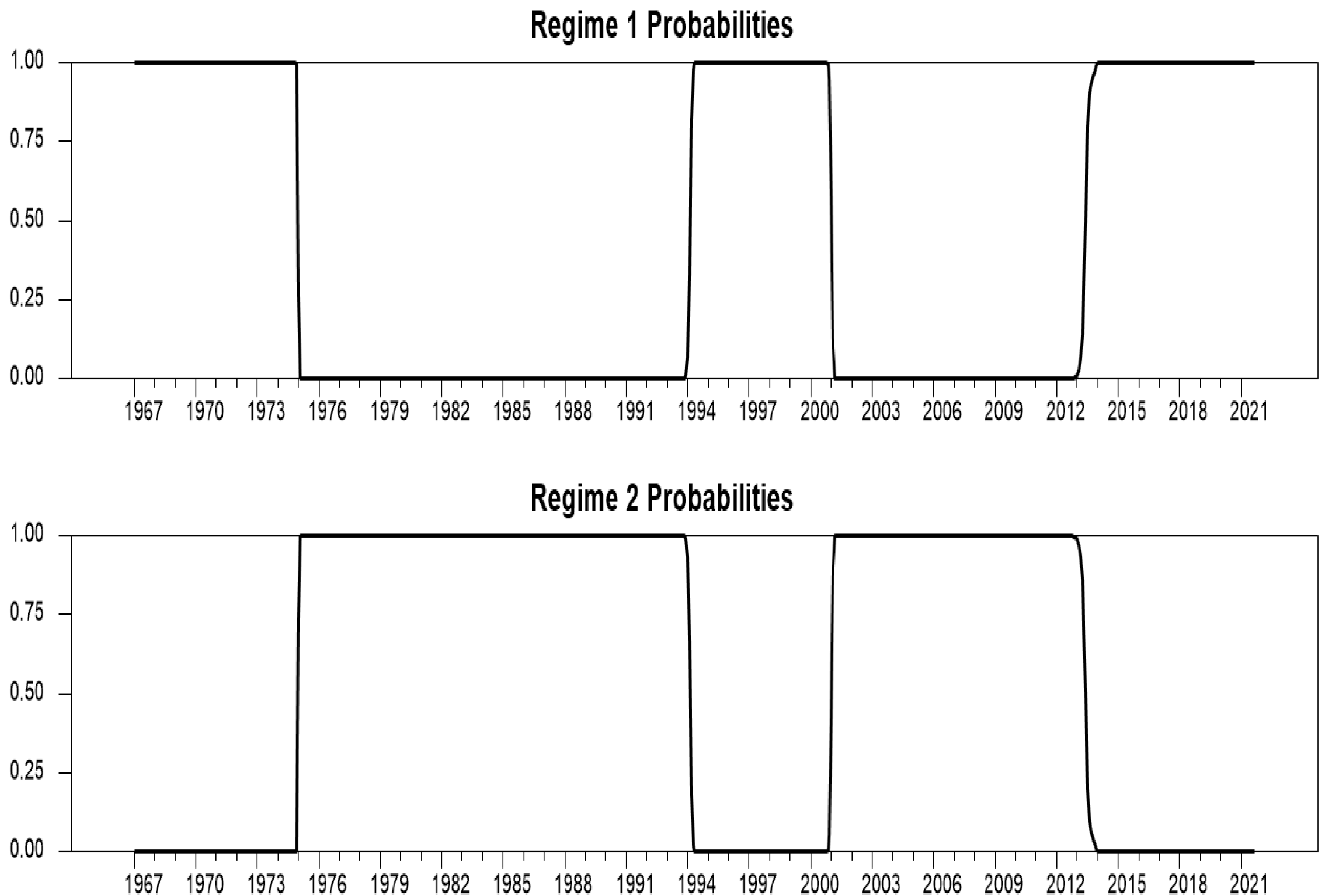
Appendix Table A1. Parameter estimates of the Markov-switching NQ Model

Goods:
1 = Consumption, c
2 = Leisure, ℓ
3 = m
4 = a

Coefficient	Divisia M1		Divisia M2		Divisia M3		Divisia M4	
	Regime 1	Regime 2	Regime 1	Regime 2	Regime 1	Regime 2	Regime 1	Regime 2
θ_1	−0.2123 (0.0002)	−0.2181 (0.0003)	−0.2059 (0.0004)	−0.2133 (0.0008)	−0.2085 (0.0005)	−0.2116 (0.0006)	−0.2069 (0.0004)	−0.2152 (0.0003)
θ_2	0.2004 (0.0002)	0.2114 (0.0002)	0.2013 (0.0005)	0.2140 (0.0003)	0.2034 (0.0005)	0.2103 (0.0004)	0.2019 (0.0004)	0.2099 (0.0004)
θ_3	0.0096 (0.0001)	0.0055 (0.0001)	0.0063 (0.0001)	0.0028 (0.0001)	0.0059 (0.0001)	0.0037 (0.0002)	0.0050 (0.0000)	0.0053 (0.0001)
θ_4	0.0023 (0.0000)	0.0012 (0.0000)	−0.0018 (0.0000)	−0.0035 (0.0001)	−0.0008 (0.0001)	−0.0024 (0.0001)		
b_1	0.9880 (0.0002)	0.9908 (0.0002)	0.9848 (0.0003)	0.9876 (0.0005)	0.9866 (0.0005)	0.9867 (0.0004)	0.9875 (0.0004)	0.9915 (0.0003)
b_2	0.0116 (0.0001)	0.0067 (0.0001)	0.0121 (0.0004)	0.0068 (0.0002)	0.0109 (0.0005)	0.0089 (0.0003)	0.0120 (0.0004)	0.0083 (0.0003)
b_3	0.0003 (0.0000)	0.0018 (0.0001)	0.0003 (0.0001)	0.0014 (0.0001)	0.0002 (0.0000)	0.0010 (0.0001)	0.0005 (0.0000)	0.0002 (0.0001)
b_4	0.0002 (0.0000)	0.0007 (0.0000)	0.0029 (0.0000)	0.0043 (0.0001)	0.0022 (0.0001)	0.0035 (0.0001)		
β_{11}	−0.0032 (0.0012)	−0.0213 (0.0030)	−0.0117 (0.0023)	−0.0187 (0.0034)	−0.0146 (0.0063)	−0.0184 (0.0036)	−0.0122 (0.0036)	−0.0121 (0.0027)
β_{12}	0.0018 (0.0010)	0.0169 (0.0029)	0.0178 (0.0026)	0.0176 (0.0029)	0.0149 (0.0059)	0.0187 (0.0029)	0.0127 (0.0035)	0.0118 (0.0023)
β_{13}	0.0028 (0.0005)	0.0064 (0.0005)	−0.0063 (0.0006)	0.0030 (0.0004)	−0.0009 (0.0005)	0.0030 (0.0007)	−0.0005 (0.0003)	0.0003 (0.0004)
β_{14}	−0.0014 (0.0003)	−0.0020 (0.0002)	0.0003 (0.0002)	−0.0019 (0.0007)	0.0006 (0.0008)	−0.0032 (0.0007)		
β_{22}	−0.0090 (0.0010)	−0.0153 (0.0026)	−0.0271 (0.0029)	−0.0166 (0.0027)	−0.0155 (0.0057)	−0.0208 (0.0030)	−0.0132 (0.0035)	−0.0141 (0.0022)
β_{23}	0.0059 (0.0004)	−0.0034 (0.0006)	0.0096 (0.0007)	−0.0027 (0.0004)	0.0009 (0.0006)	−0.0019 (0.0008)	0.0005 (0.0003)	0.0022 (0.0004)
β_{24}	0.0013 (0.0003)	0.0018 (0.0003)	−0.0004 (0.0003)	0.0017 (0.0007)	−0.0003 (0.0009)	0.0040 (0.0009)		
β_{33}	−0.0095 (0.0004)	−0.0035 (0.0003)	−0.0035 (0.0005)	−0.0010 (0.0009)	−0.0001 (0.0005)	−0.0012 (0.0006)	−0.0000 (0.0001)	−0.0025 (0.0003)
β_{34}	0.0008 (0.0001)	0.0005 (0.0001)	0.0002 (0.0003)	0.0007 (0.0007)	0.0000 (0.0003)	0.0001 (0.0004)		
β_{44}	−0.0007 (0.0000)	−0.0002 (0.0001)	−0.0001 (0.0003)	−0.0004 (0.0007)	−0.0004 (0.0005)	−0.0009 (0.0006)		

Note: Numbers in parentheses are standard errors.

Appendix Figure A1. Probabilities of regimes based on the Markov-switching NQ demand system and Divisia M1



Appendix Table A2. Income and price elasticities

Good i	A. Income	B. Own- and cross-price			
	η_i	$\eta_{i,c}$	$\eta_{i,\ell}$	$\eta_{i,m}$	$\eta_{i,a}$
			Regime 1		
c	1.1841 (0.0602)	−0.9865 (0.0015)	−0.1852 (0.0542)	−0.0077 (0.0045)	−0.0048 (0.0015)
ℓ	0.0938 (0.0249)	−0.0762 (0.0227)	−0.0287 (0.0052)	0.0062 (0.0023)	0.0049 (0.0041)
m (Divisia M1)	0.1256 (0.1722)	−0.0417 (0.1692)	0.1543 (0.0213)	−0.2556 (0.0859)	0.0174 (0.0755)
a	0.0616 (0.0645)	−0.1902 (0.0883)	0.2270 (0.1887)	0.0749 (0.1050)	−0.1733 (0.0757)
			Regime 2		
c	1.1791 (0.0387)	−0.9923 (0.0020)	−0.1746 (0.0347)	−0.0063 (0.0039)	−0.0059 (0.0019)
ℓ	0.0530 (0.0125)	−0.0233 (0.0084)	−0.0302 (0.0081)	−0.0050 (0.0010)	0.0055 (0.0016)
m (Divisia M1)	0.3580 (0.0817)	−0.0926 (0.0568)	−0.1814 (0.0459)	−0.1245 (0.0268)	0.0406 (0.0195)
a	0.5041 (0.0508)	−0.6506 (0.0576)	0.1434 (0.0536)	0.0703 (0.0428)	−0.0672 (0.0148)

Note: Numbers in parentheses are standard errors of the time series elasticities.

Appendix Table A3. Allen elasticities of substitution

Good i	$\sigma^a_{i,c}$	$\sigma^a_{i,\ell}$	$\sigma^a_{i,m}$	$\sigma^a_{i,a}$
		Regime 1		
c	−0.0007 (0.0003)	0.0036 (0.0025)	0.0818 (0.0281)	−0.1656 (0.0380)
ℓ		−0.1076 (0.0482)	1.1854 (0.0490)	1.7819 (1.6353)
m (Divisia M1)			−47.2469 (19.4351)	0.9971 (26.6450)
a				−55.7964 (36.7753)
		Regime 2		
c	−0.0050 (0.0010)	0.0254 (0.0066)	0.2492 (0.0441)	−0.2705 (0.0490)
ℓ		−0.1596 (0.0721)	−0.9138 (0.3944)	1.5080 (0.4946)
m (Divisia M1)			−22.9588 (8.4313)	10.2193 (3.4748)
a				−16.7477 (3.6551)

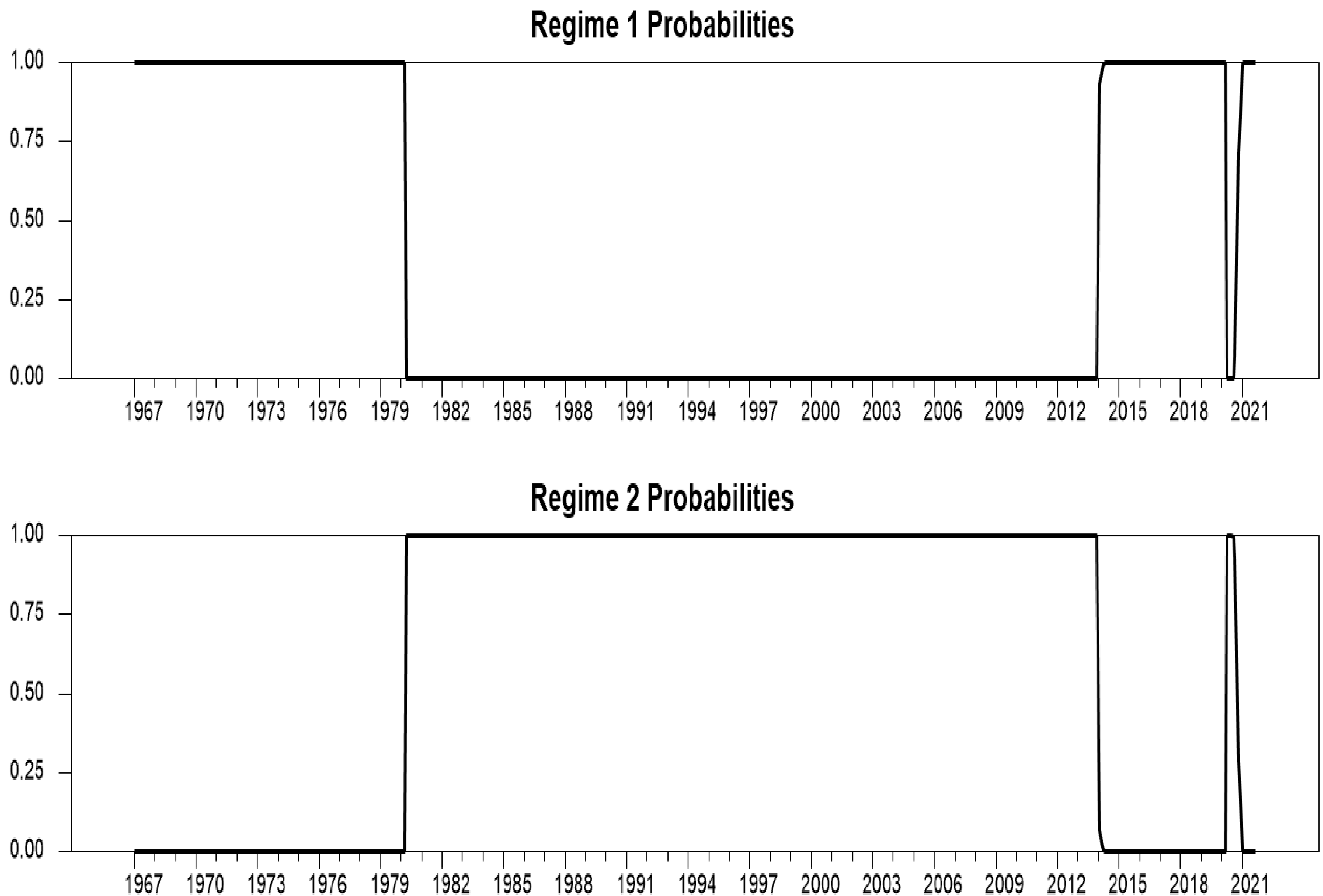
Note: Numbers in parentheses are standard errors of the time series elasticities.

Appendix Table A4. Morishima elasticities of substitution

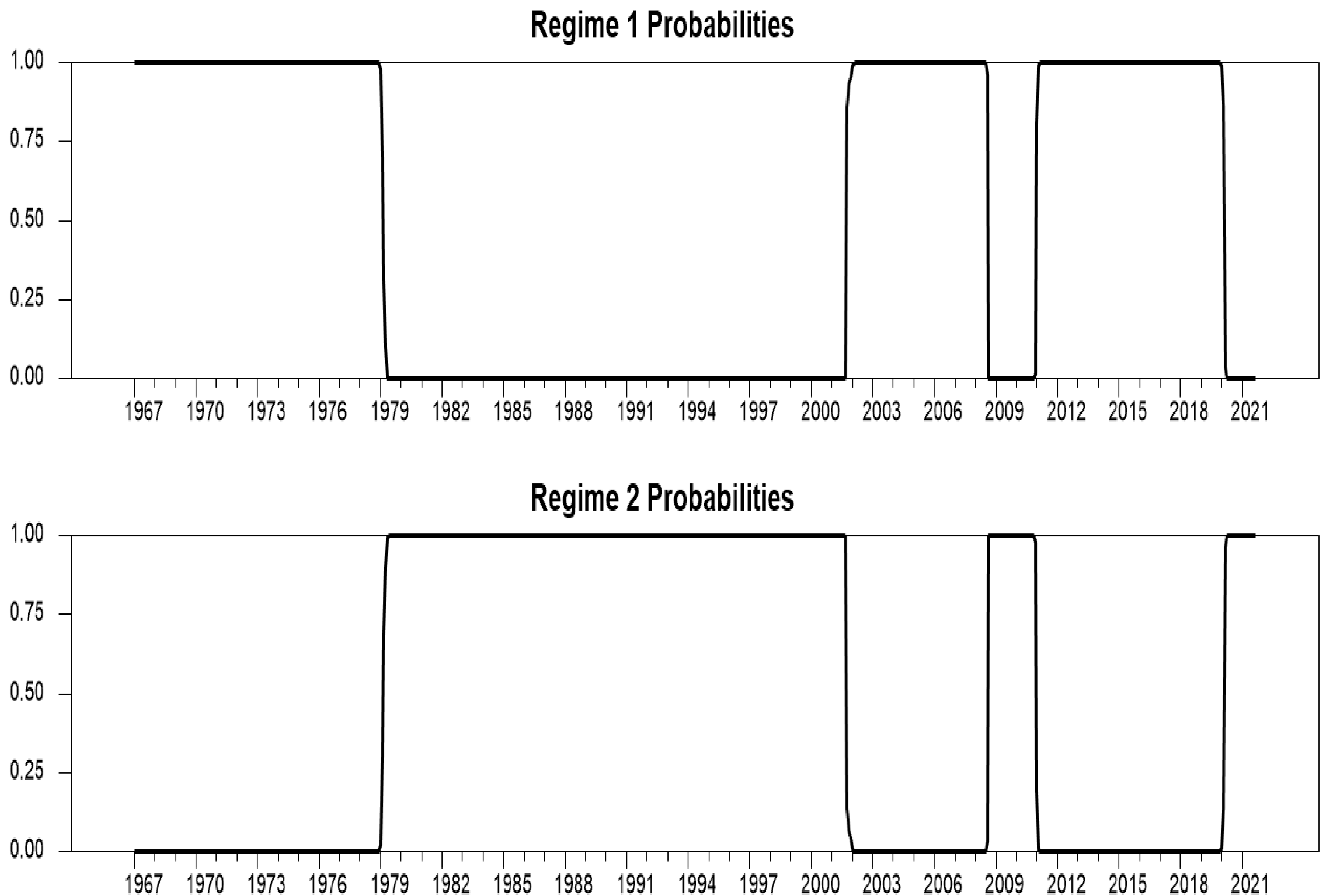
Good i	$\sigma^m_{i,c}$	$\sigma^m_{i,\ell}$	$\sigma^m_{i,m}$	$\sigma^m_{i,a}$
		Regime 1		
c		0.0156 (0.0045)	0.2560 (0.0865)	0.1725 (0.0757)
ℓ	0.0037 (0.0022)		0.2621 (0.0886)	0.1783 (0.0795)
m (Divisia M1)	0.0685 (0.0234)	0.1848 (0.0340)		0.1908 (0.0521)
a	−0.1379 (0.0344)	0.2514 (0.1964)	0.3306 (0.1857)	
		Regime 2		
c		0.0262 (0.0080)	0.1239 (0.0267)	0.0641 (0.0141)
ℓ	0.0256 (0.0063)		0.1177 (0.0254)	0.0709 (0.0151)
m (Divisia M1)	0.2139 (0.0413)	−0.1068 (0.0337)		0.1071 (0.0298)
a	−0.2223 (0.0377)	0.2406 (0.0584)	0.1959 (0.0676)	

Note: Numbers in parentheses are standard errors of the timer series elasticities.

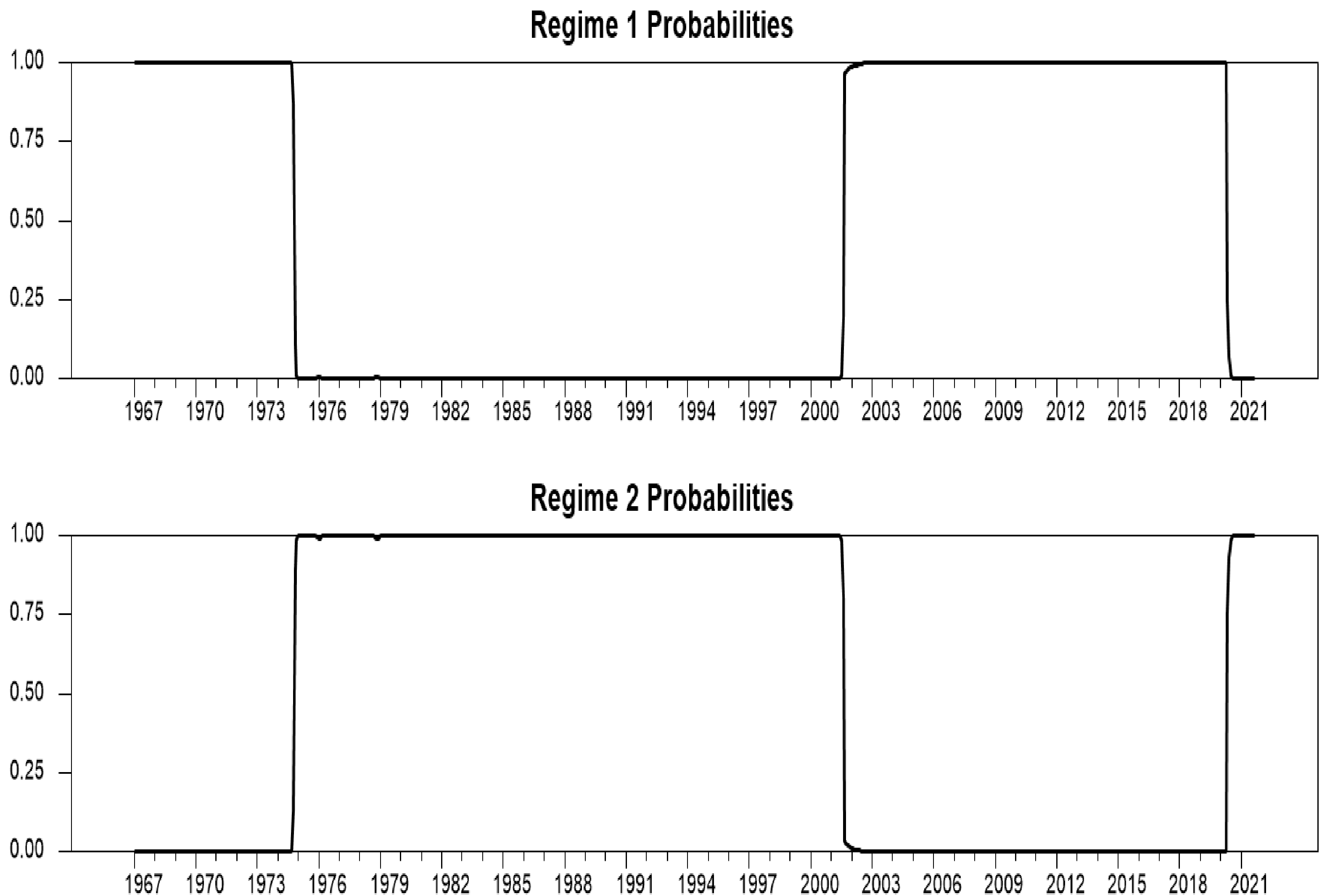
Appendix Figure A2. Probabilities of regimes based on the Markov-switching NQ demand system and Divisia M2



Appendix Figure A3. Probabilities of regimes based on the Markov-switching NQ demand system and Divisia M3



Appendix Figure A4. Probabilities of regimes based on the Markov-switching NQ demand system and Divisia M4



Appendix Table A5. Income and Price Elasticities

Regime 1		Income	Own- and cross-price			
Good i	m	η_i	$\eta_{i,c}$	$\eta_{i,\ell}$	$\eta_{i,m}$	$\eta_{i,a}$
c	Divisia M1	1.1841 (0.0602)	−0.9865 (0.0015)	−0.1852 (0.0542)	−0.0077 (0.0045)	−0.0048 (0.0015)
	Divisia M2	1.2030 (0.0557)	−0.9850 (0.0021)	−0.2047 (0.0532)	−0.0108 (0.0036)	−0.0025 (0.0007)
	Divisia M3	1.1824 (0.0577)	−0.9884 (0.0014)	−0.1837 (0.0552)	−0.0083 (0.0029)	−0.0021 (0.0007)
	Divisia M4	1.1699 (0.0568)	−0.9882 (0.0012)	−0.1737 (0.0548)	−0.0080 (0.0019)	
ℓ	Divisia M1	0.0938 (0.0249)	−0.0762 (0.0227)	−0.0287 (0.0052)	0.0062 (0.0023)	0.0049 (0.0041)
	Divisia M2	0.0848 (0.0234)	−0.0437 (0.0148)	−0.0603 (0.0201)	0.0193 (0.0076)	−0.0001 (0.0027)
	Divisia M3	0.0847 (0.0240)	−0.0478 (0.0166)	−0.0405 (0.0107)	0.0035 (0.0022)	0.0001 (0.0005)
	Divisia M4	0.0887 (0.0234)	−0.0545 (0.0187)	−0.0539 (0.0118)	0.0197 (0.0062)	
m	Divisia M1	0.1256 (0.1722)	−0.0417 (0.1692)	0.1543 (0.0213)	−0.2556 (0.0859)	0.0174 (0.0755)
	Divisia M2	0.0932 (0.1364)	−0.3879 (0.2600)	0.5232 (0.2329)	−0.2300 (0.0944)	0.0015 (0.0304)
	Divisia M3	0.0699 (0.0279)	−0.1448 (0.0687)	0.0902 (0.0543)	−0.0154 (0.0107)	0.0001 (0.0029)
	Divisia M4	0.1757 (0.0929)	−0.4307 (0.1795)	0.5151 (0.1704)	−0.2601 (0.0892)	
a	Divisia M1	0.0616 (0.0645)	−0.1902 (0.0883)	0.2270 (0.1887)	0.0749 (0.1050)	−0.1733 (0.0757)
	Divisia M2	1.8556 (0.3810)	−1.4839 (0.1915)	−0.3696 (0.2815)	0.0278 (0.0753)	−0.0299 (0.0136)
	Divisia M3	1.3574 (0.1492)	−1.1016 (0.0365)	−0.2005 (0.1144)	−0.0079 (0.0086)	−0.0475 (0.0136)
	Divisia M4					

Note: Numbers in parentheses are standard error of the time series elasticities.

Appendix Table A5. Income and Price Elasticities (cont.)

Regime 2		Income	Own- and cross-price			
Good i	m	η_i	$\eta_{i,c}$	$\eta_{i,\ell}$	$\eta_{i,m}$	$\eta_{i,a}$
c	Divisia M1	1.1791 (0.0387)	−0.9923 (0.0020)	−0.1746 (0.0347)	−0.0063 (0.0039)	−0.0059 (0.0019)
	Divisia M2	1.1586 (0.0256)	−0.9899 (0.0027)	−0.1589 (0.0248)	−0.0056 (0.0026)	−0.0043 (0.0012)
	Divisia M3	1.1678 (0.0265)	−0.9898 (0.0030)	−0.1677 (0.0257)	−0.0066 (0.0028)	−0.0037 (0.0010)
	Divisia M4	1.1786 (0.0328)	−0.9896 (0.0017)	−0.1798 (0.0305)	−0.0093 (0.0026)	
ℓ	Divisia M1	0.0530 (0.0125)	−0.0233 (0.0084)	−0.0302 (0.0081)	−0.0050 (0.0010)	0.0055 (0.0016)
	Divisia M2	0.0486 (0.0077)	−0.0146 (0.0039)	−0.0333 (0.0085)	−0.0050 (0.0007)	0.0042 (0.0016)
	Divisia M3	0.0620 (0.0074)	−0.0254 (0.0066)	−0.0406 (0.0107)	−0.0029 (0.0015)	0.0069 (0.0014)
	Divisia M4	0.0786 (0.0113)	−0.0491 (0.0098)	−0.0451 (0.0065)	0.0157 (0.0027)	
m	Divisia M1	0.3580 (0.0817)	−0.0926 (0.0568)	−0.1814 (0.0459)	−0.1245 (0.0268)	0.0406 (0.0195)
	Divisia M2	0.4473 (0.0527)	−0.2275 (0.0460)	−0.1934 (0.0385)	−0.0809 (0.0065)	0.0545 (0.0142)
	Divisia M3	0.2676 (0.0517)	−0.0929 (0.0705)	−0.0921 (0.0469)	−0.0934 (0.0114)	0.0108 (0.0148)
	Divisia M4	0.0959 (0.1106)	−0.3092 (0.2033)	0.4229 (0.1413)	−0.2096 (0.0528)	
a	Divisia M1	0.5041 (0.0508)	−0.6506 (0.0576)	0.1434 (0.0536)	0.0703 (0.0428)	−0.0672 (0.0148)
	Divisia M2	1.8573 (0.2956)	−1.8091 (0.2249)	−0.0622 (0.0872)	0.0924 (0.0280)	−0.0783 (0.0197)
	Divisia M3	1.7583 (0.2461)	−1.8695 (0.1606)	0.1991 (0.1225)	0.0255 (0.0310)	−0.1134 (0.0242)
	Divisia M4					

Note: Numbers in parentheses are standard error of the time series elasticities.

Appendix Table A6. Allen elasticities of substitution

Regime 1 Good i	m	$\sigma^a_{i,c}$	$\sigma^a_{i,\ell}$	$\sigma^a_{i,m}$	$\sigma^a_{i,a}$
c	Divisia M1	−0.0007 (0.0003)	0.0036 (0.0025)	0.0818 (0.0281)	−0.1656 (0.0380)
	Divisia M2	−0.0036 (0.0008)	0.0319 (0.0123)	−0.3688 (0.1543)	0.0290 (0.1239)
	Divisia M3	−0.0044 (0.0007)	0.0283 (0.0078)	−0.1004 (0.0481)	0.0329 (0.0532)
	Divisia M4	−0.0016 (0.0008)	0.0196 (0.0112)	−0.2730 (0.1763)	
ℓ	Divisia M1		−0.1076 (0.0482)	1.1854 (0.0490)	1.7819 (1.6353)
	Divisia M2		−0.3129 (0.2110)	3.6255 (2.5533)	−0.0121 (1.2112)
	Divisia M3		−0.2039 (0.1059)	0.7493 (0.5166)	0.1747 (0.3171)
	Divisia M4		−0.3102 (0.1255)	4.0342 (1.8229)	
m	Divisia M1			−47.2469 (19.4351)	0.9971 (24.6450)
	Divisia M2			−42.5882 (31.0907)	1.4166 (13.5807)
	Divisia M3			−2.9167 (2.4518)	−0.2656 (1.9260)
	Divisia M4			−53.4119 (28.0647)	
a	Divisia M1				−55.7964 (36.7753)
	Divisia M2				−12.9774 (5.7673)
	Divisia M3				−25.9002 (6.6853)
	Divisia M4				

Note: Numbers in parentheses are standard errors of the time series elasticities.

Appendix Table A6. Allen elasticities of substitution (cont.)

Regime 2 Good i	m	$\sigma^a_{i,c}$	$\sigma^a_{i,\ell}$	$\sigma^a_{i,m}$	$\sigma^a_{i,a}$
c	Divisia M1	−0.0050 (0.0010)	0.0254 (0.0066)	0.2492 (0.0441)	−0.2705 (0.0490)
	Divisia M2	−0.0053 (0.0007)	0.0315 (0.0074)	0.1807 (0.0340)	−0.2758 (0.0394)
	Divisia M3	−0.0054 (0.0010)	0.0319 (0.0101)	0.1585 (0.0435)	−0.4619 (0.0666)
	Divisia M4	−0.0024 (0.0015)	0.0251 (0.0068)	−0.3403 (0.1584)	
ℓ	Divisia M1		−0.1596 (0.0721)	−0.9138 (0.3944)	1.5080 (0.4946)
	Divisia M2		−0.1970 (0.0744)	−0.9593 (0.3175)	1.4717 (0.2916)
	Divisia M3		−0.2242 (0.0981)	−0.3523 (0.3742)	3.2021 (0.7612)
	Divisia M4		−0.2228 (0.0732)	2.9550 (1.4113)	
m	Divisia M1			−22.9588 (8.4313)	10.2193 (3.4748)
	Divisia M2			−16.4447 (6.5424)	19.0981 (2.7822)
	Divisia M3			−17.2650 (10.5434)	3.9113 (7.9259)
	Divisia M4			−40.2189 (27.5778)	
a	Divisia M1				−16.7477 (3.6551)
	Divisia M2				−25.7961 (7.1636)
	Divisia M3				−48.7798 (8.6426)
	Divisia M4				

Note: Numbers in parentheses are standard errors of the time series elasticities.

Appendix Table A7. Morishima elasticities of substitution

Regime 1 Good i	m	$\sigma^{m}_{i,c}$	$\sigma^{m}_{i,\ell}$	$\sigma^{m}_{i,m}$	$\sigma^{m}_{i,a}$
c	Divisia M1		0.0156 (0.0045)	0.2560 (0.0865)	0.1725 (0.0757)
	Divisia M2		0.0516 (0.0206)	0.2275 (0.0939)	0.0262 (0.0130)
	Divisia M3		0.0323 (0.0102)	0.0145 (0.0104)	0.0451 (0.0130)
	Divisia M4		0.0182 (0.0105)	−0.2326 (0.1543)	
ℓ	Divisia M1	0.0037 (0.0022)		0.2621 (0.0886)	0.1783 (0.0795)
	Divisia M2	0.0295 (0.0120)		0.2495 (0.1016)	0.0262 (0.0121)
	Divisia M3	0.0276 (0.0076)		0.0190 (0.0127)	0.0452 (0.0126)
	Divisia M4	0.0442 (0.0119)		0.5814 (0.1886)	
m	Divisia M1	0.0685 (0.0234)	0.1848 (0.0340)		0.1908 (0.0521)
	Divisia M2	−0.3042 (0.1427)	0.5816 (0.2678)		0.0278 (0.0365)
	Divisia M3	−0.0817 (0.0436)	0.1283 (0.0660)		0.0453 (0.0150)
	Divisia M4	0.2580 (0.0885)	0.2794 (0.0946)		
a	Divisia M1	−0.1379 (0.0344)	0.2514 (0.1964)	0.3306 (0.1857)	
	Divisia M2	0.0229 (0.1030)	0.0112 (0.1938)	0.2712 (0.0854)	
	Divisia M3	0.0293 (0.0429)	0.0465 (0.0524)	0.0160 (0.0043)	
	Divisia M4				

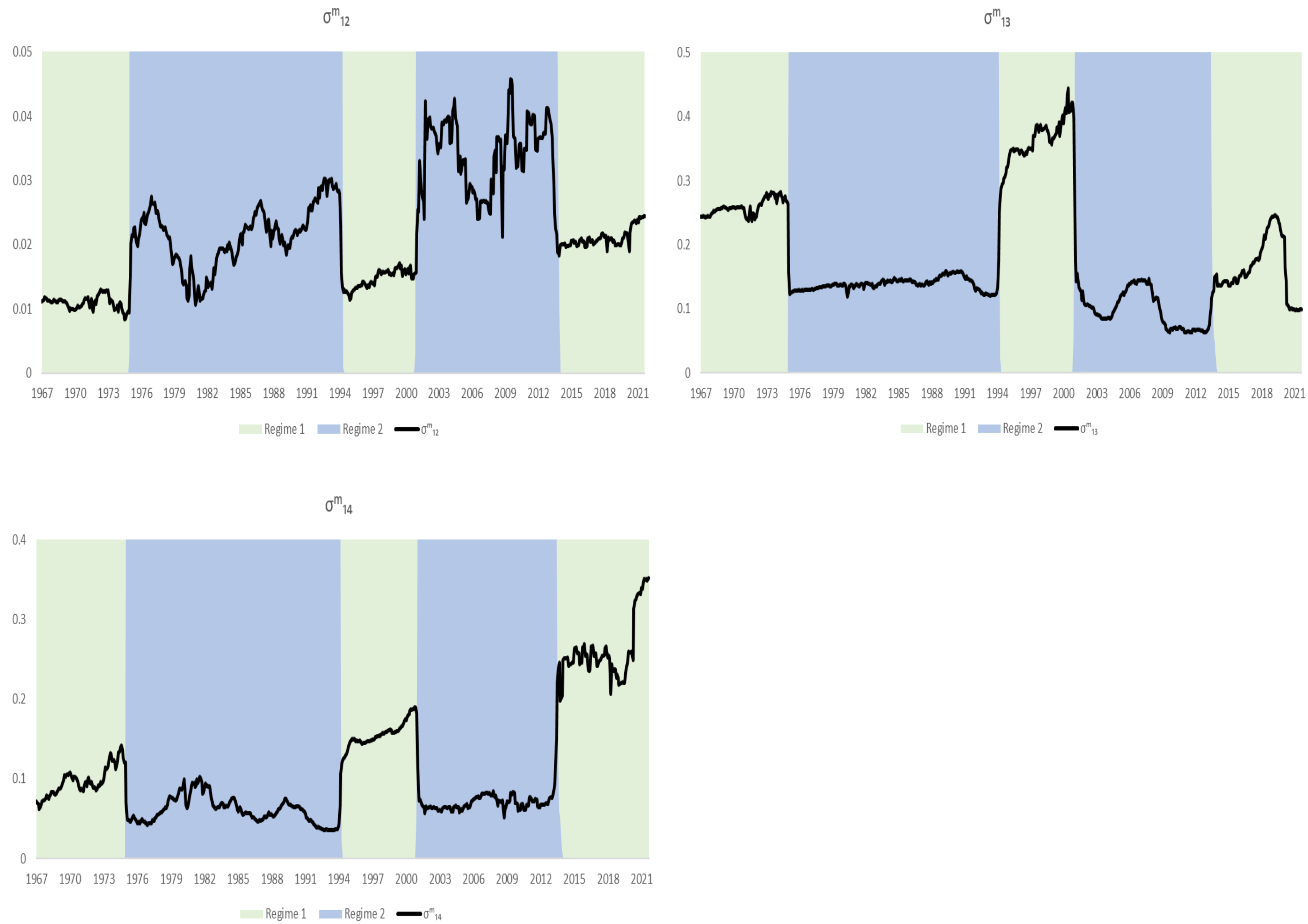
Note: Numbers in parentheses are standard errors of the time series elasticities.

Appendix Table A7. Morishima elasticities of substitution (cont.)

Regime 2 Good i	m	$\sigma^m_{i,c}$	$\sigma^m_{i,\ell}$	$\sigma^m_{i,m}$	$\sigma^m_{i,a}$
c	Divisia M1		0.0262 (0.0080)	0.1239 (0.0267)	0.0641 (0.0141)
	Divisia M2		0.0309 (0.0088)	0.0794 (0.0064)	0.0720 (0.0182)
	Divisia M3		0.0361 (0.0118)	0.0928 (0.0115)	0.1082 (0.0226)
	Divisia M4		0.0231 (0.0068)	−0.2856 (0.1410)	
ℓ	Divisia M1	0.0256 (0.0063)		0.1177 (0.0254)	0.0709 (0.0151)
	Divisia M2	0.0314 (0.0072)		0.0738 (0.0058)	0.0772 (0.0182)
	Divisia M3	0.0316 (0.0099)		0.0892 (0.0127)	0.1163 (0.0232)
	Divisia M4	0.0371 (0.0068)		0.4693 (0.1599)	
m	Divisia M1	0.2139 (0.0413)	−0.1068 (0.0337)		0.1071 (0.0298)
	Divisia M2	0.1585 (0.0306)	−0.1047 (0.0306)		0.1286 (0.0251)
	Divisia M3	0.1383 (0.0372)	−0.0219 (0.0585)		0.1206 (0.0335)
	Divisia M4	0.2073 (0.0525)	0.2253 (0.0548)		
a	Divisia M1	−0.2223 (0.0377)	0.2406 (0.0584)	0.1959 (0.0676)	
	Divisia M2	−0.2298 (0.0311)	0.2305 (0.0379)	0.1817 (0.0352)	
	Divisia M3	−0.3856 (0.0582)	0.4936 (0.0880)	0.1291 (0.0266)	
	Divisia M4				

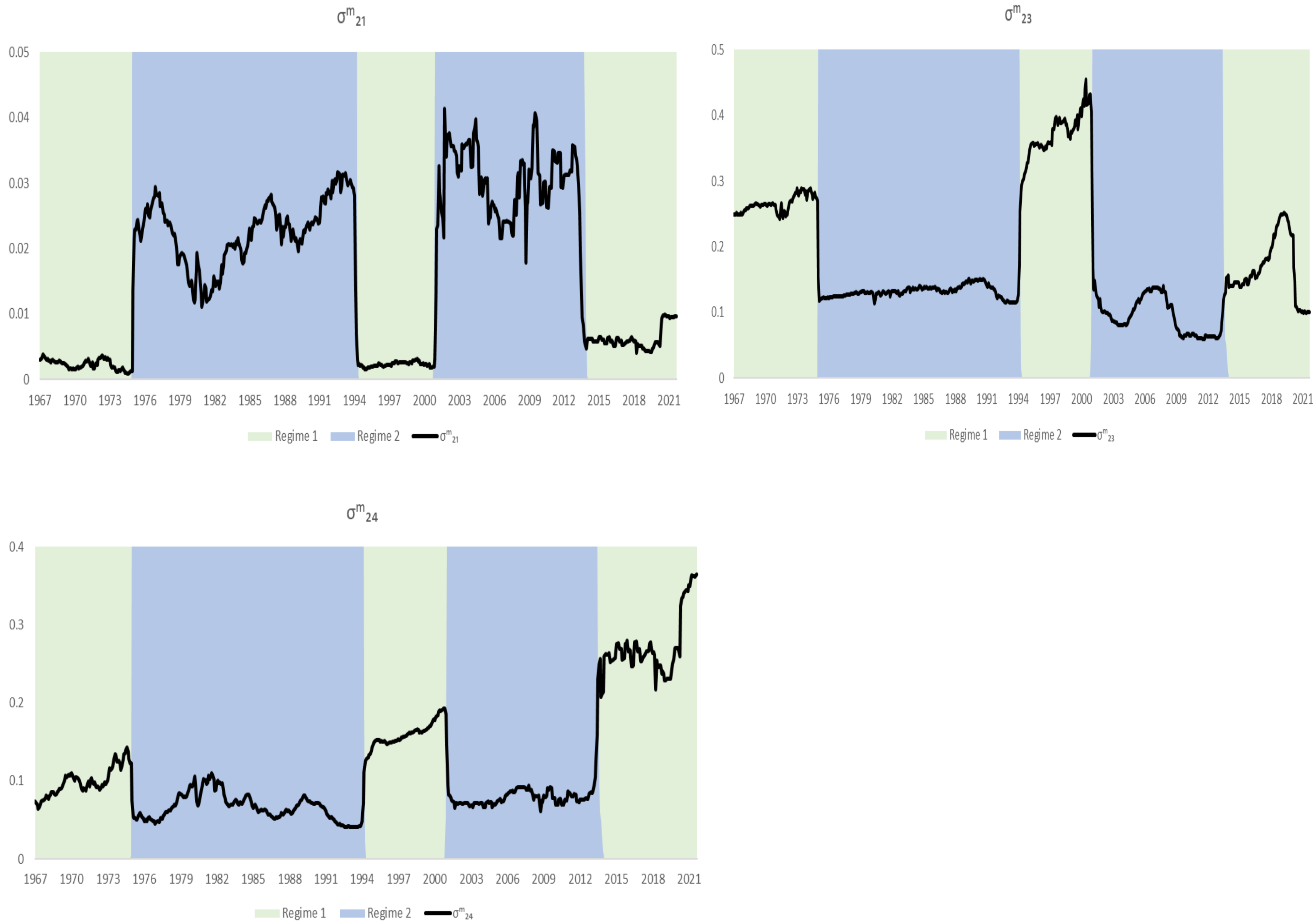
Note: Numbers in parentheses are standard errors of the time series elasticities.

Appendix Figure A5. Morishima elasticities of substitution for consumption based on the Markov-switching NQ demand system and Divisia M1



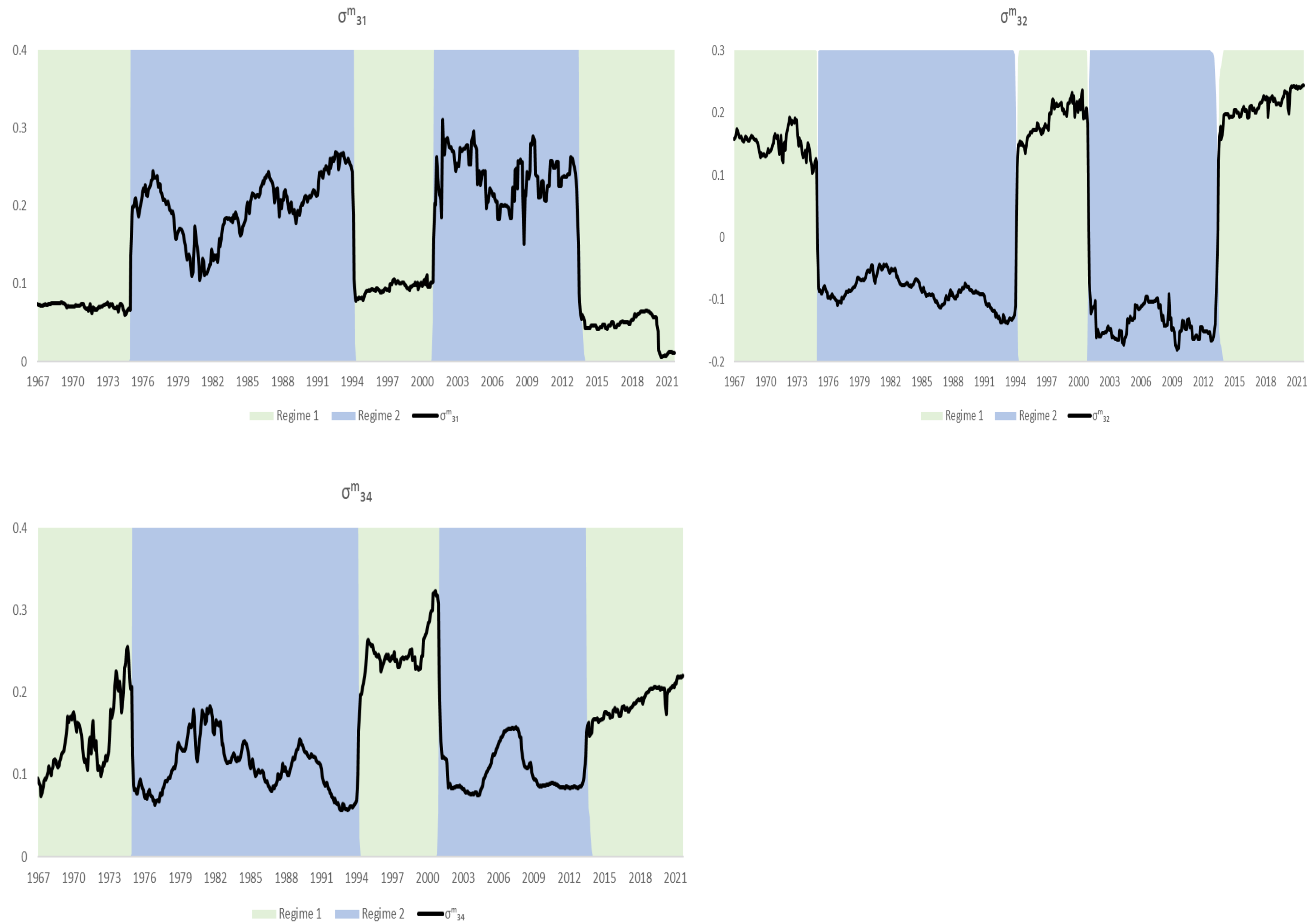
Note: 1 = Consumption, 2 = Leisure, 3 = Monetary aggregate, 4 = Aggregate of non-money monetary assets

Appendix Figure A6. Morishima elasticities of substitution for leisure based on the Markov-switching NQ demand system and Divisia M1



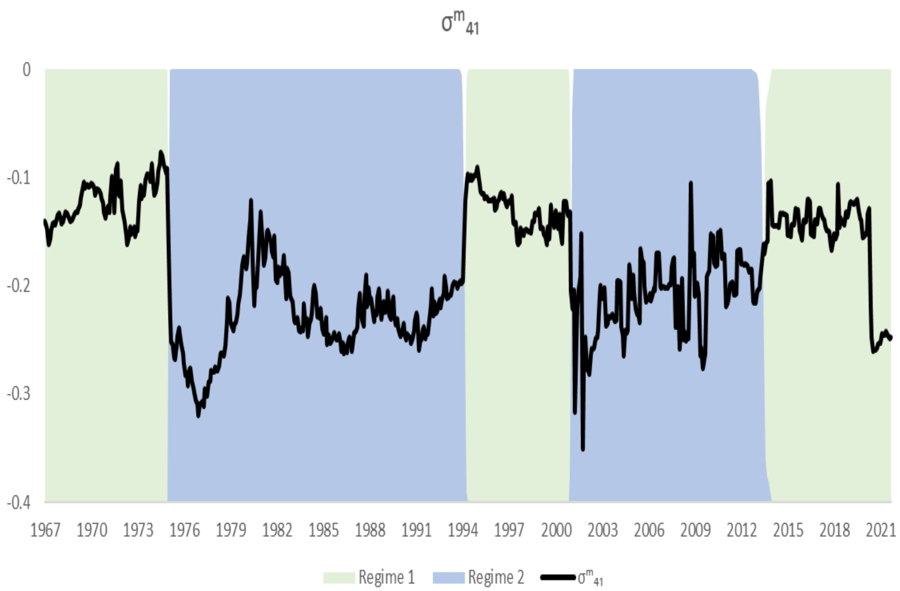
Note: 1 = Consumption, 2 = Leisure, 3 = Monetary aggregate, 4 = Aggregate of non-money monetary assets

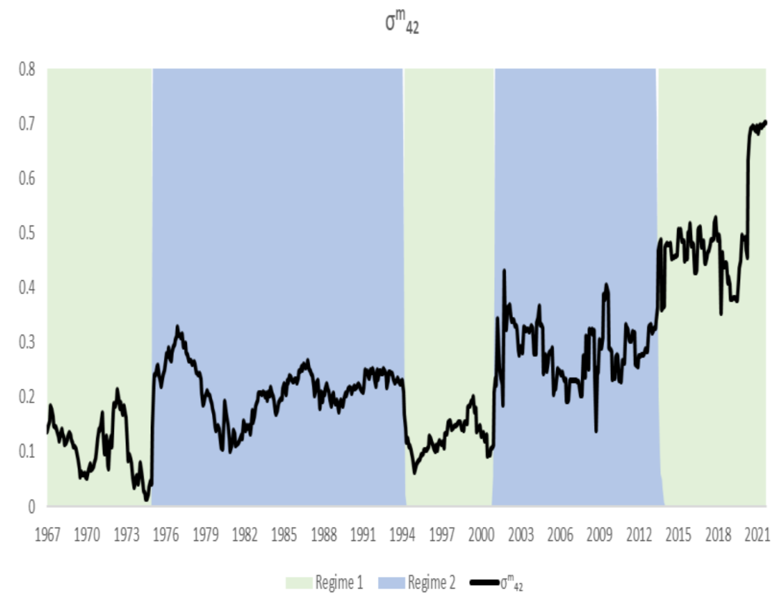
Appendix Figure A7. Morishima elasticities of substitution for money based on the Markov-switching NQ demand system and Divisia M1

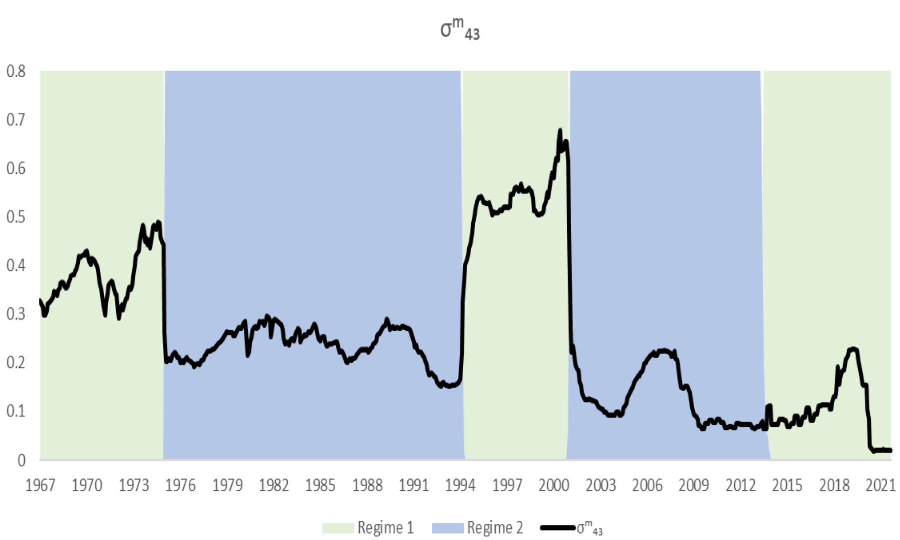


Note: 1 = Consumption, 2 = Leisure, 3 = Monetary aggregate, 4 = Aggregate of non-money monetary assets

Appendix Figure A8. Morishima elasticities of substitution for other monetary assets based on the Markov-switching NQ demand system and Divisia M1

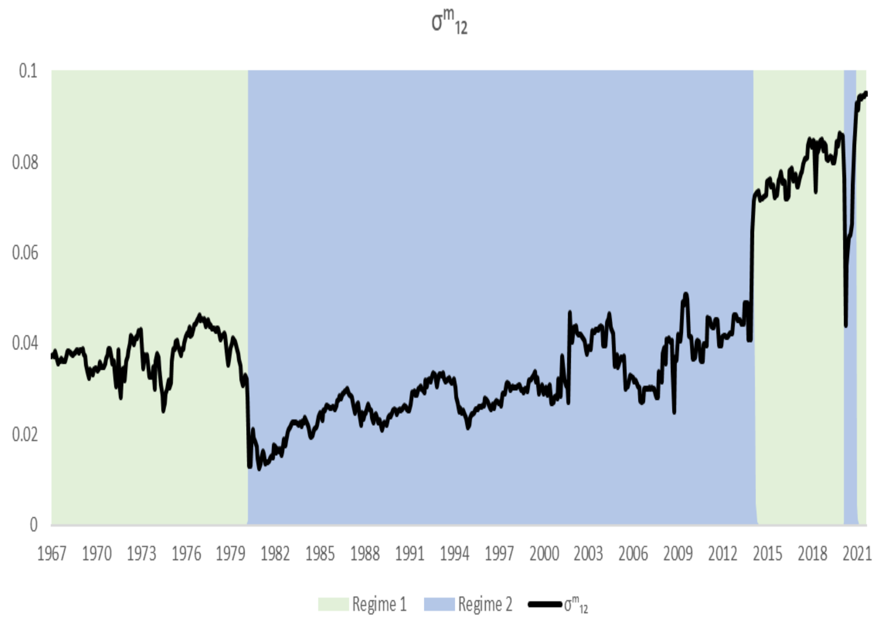


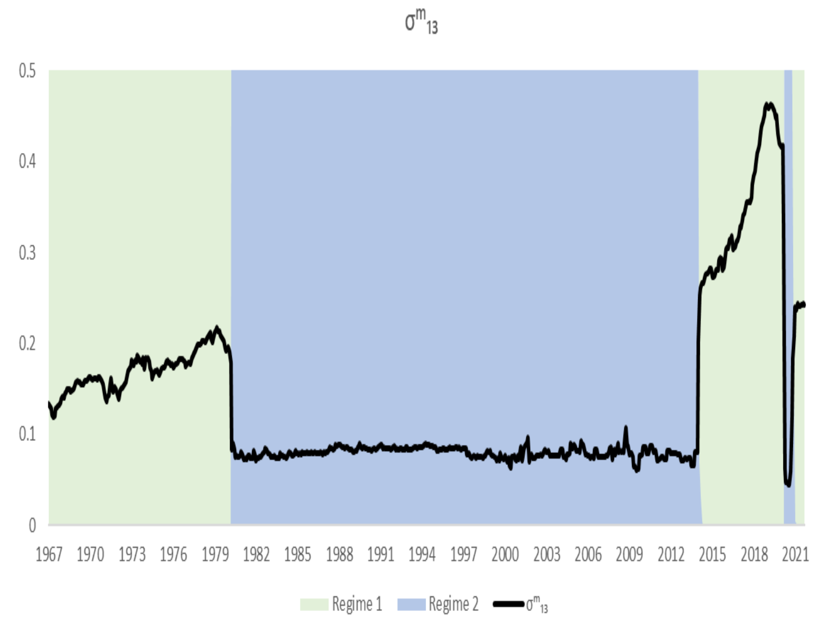


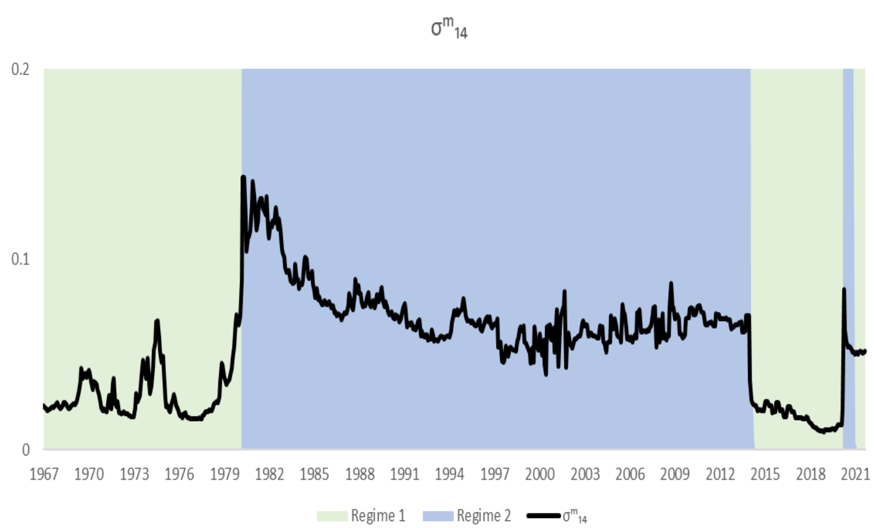


Note: 1 = Consumption, 2 = Leisure, 3 = Monetary aggregate, 4 = Aggregate of non-money monetary assets

Appendix Figure A9. Morishima elasticities of substitution for consumption based on the Markov-switching NQ demand system and Divisia M2

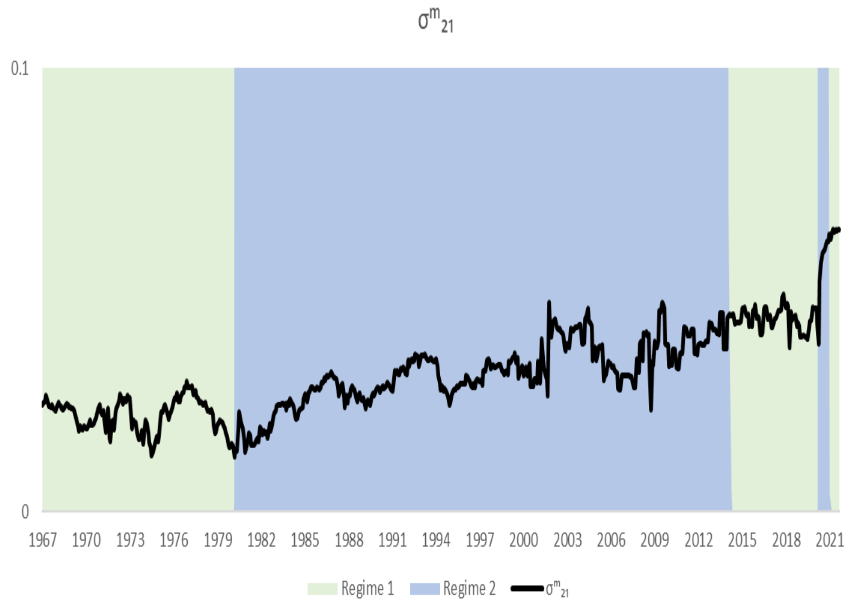


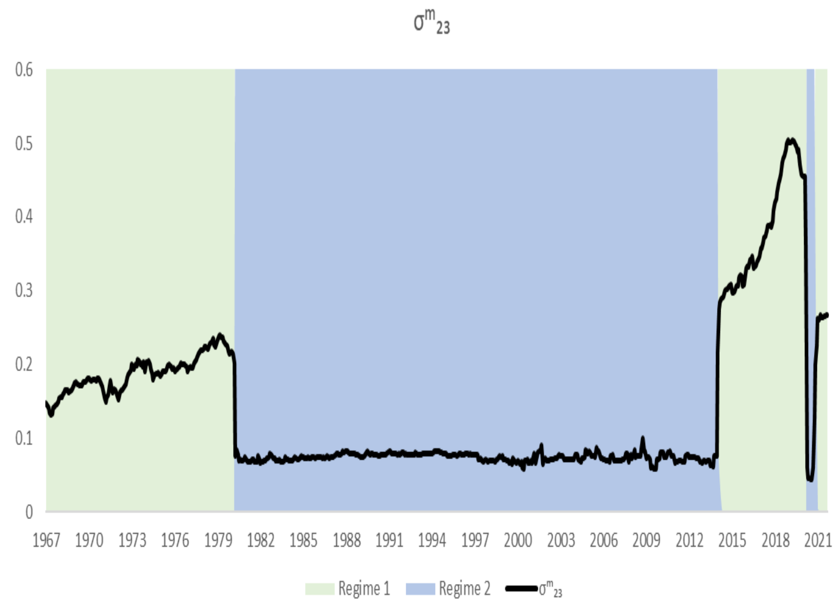


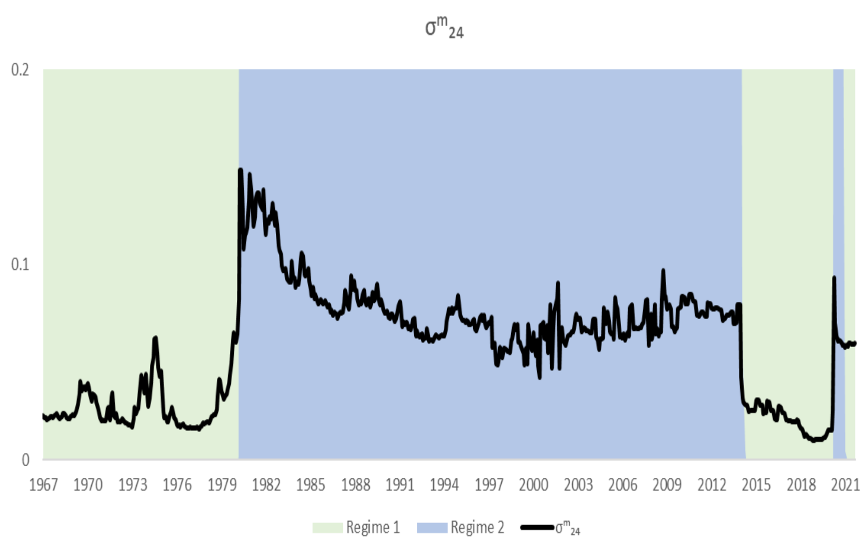


Note: 1 = Consumption, 2 = Leisure, 3 = Monetary aggregate, 4 = Aggregate of non-money monetary assets

Appendix Figure A10. Morishima elasticities of substitution for leisure based on the Markov-switching NQ demand system and Divisia M2

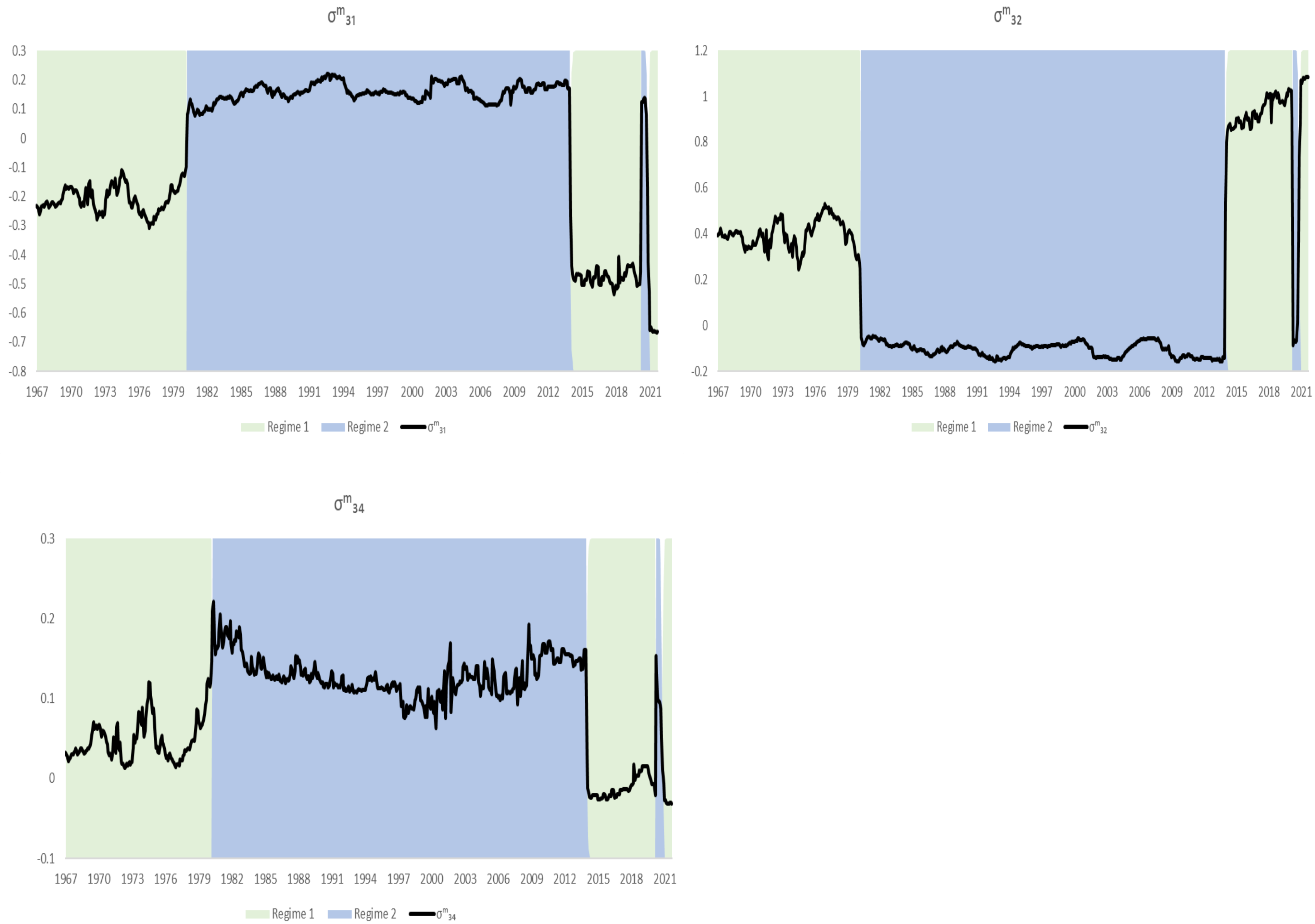






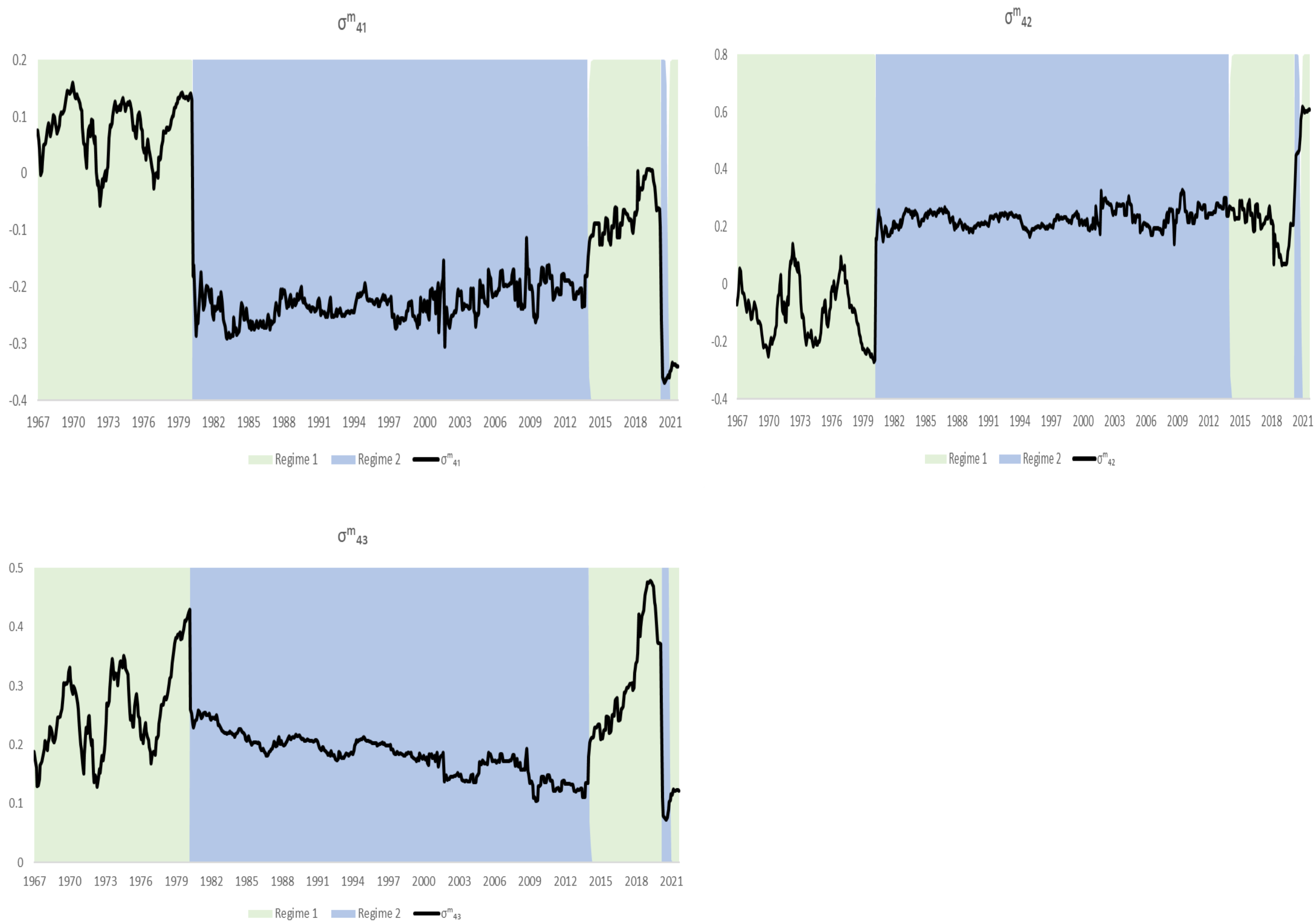
Note: 1 = Consumption, 2 = Leisure, 3 = Monetary aggregate, 4 = Aggregate of non-money monetary assets

Appendix Figure A11. Morishima elasticities of substitution for money based on the Markov-switching NQ demand system and Divisia M2



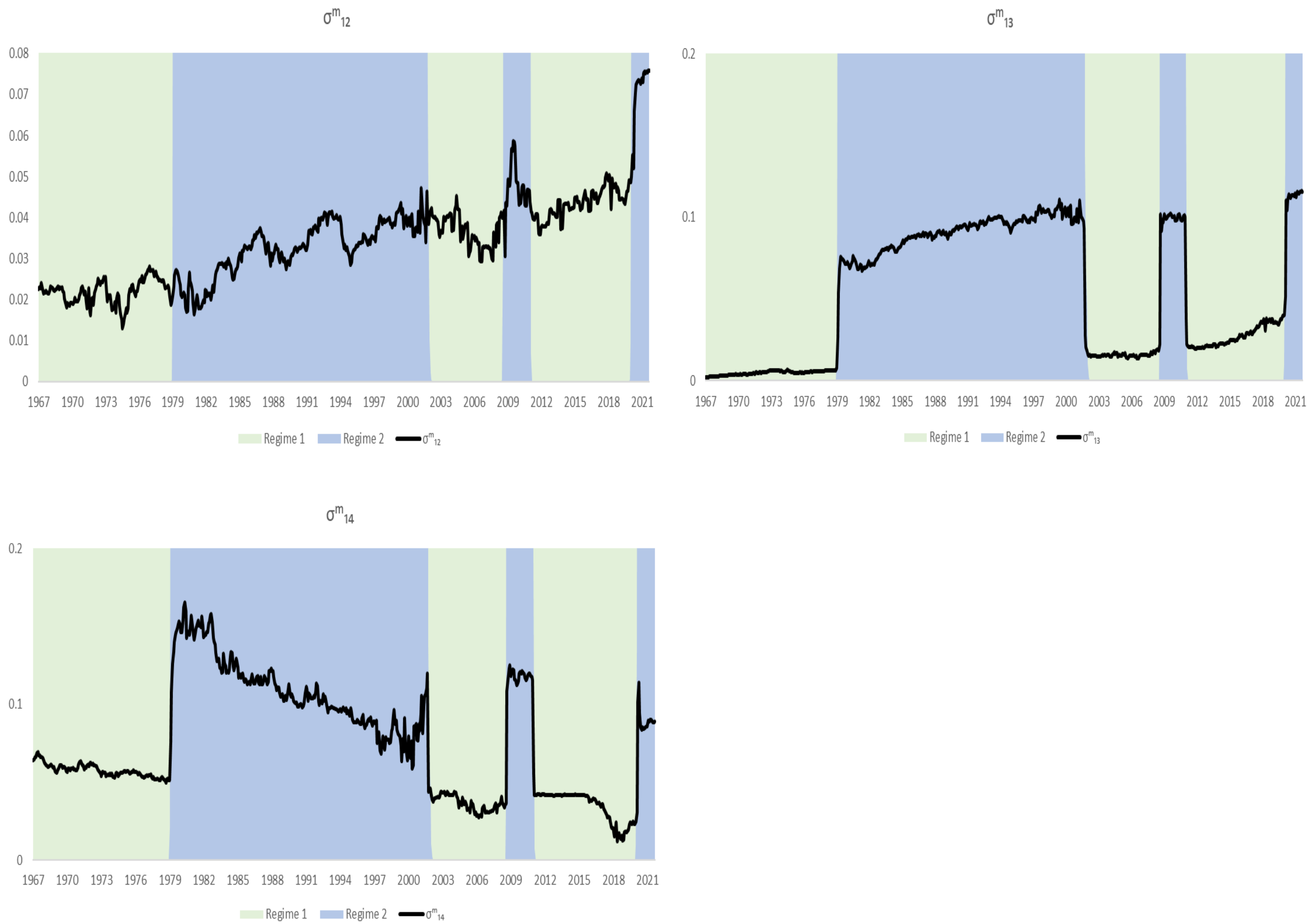
Note: 1 = Consumption, 2 = Leisure, 3 = Monetary aggregate, 4 = Aggregate of non-money monetary assets

Appendix Figure A12. Morishima elasticities of substitution for other monetary assets based on the Markov-switching NQ demand system and Divisia M2



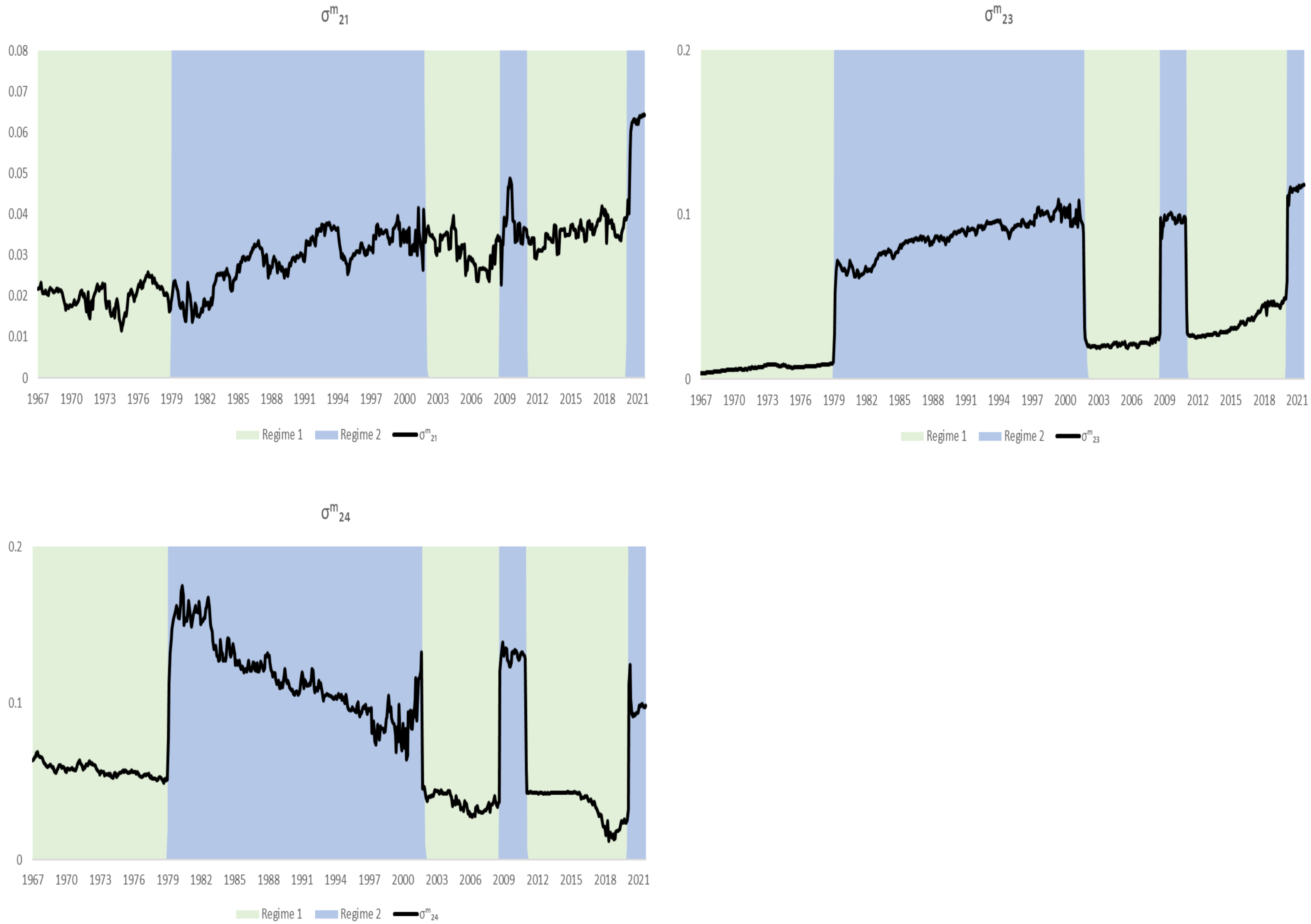
Note: 1 = Consumption, 2 = Leisure, 3 = Monetary aggregate, 4 = Aggregate of non-money monetary assets

Appendix Figure A13. Morishima elasticities of substitution for consumption based on the Markov-switching NQ demand system and Divisia M3



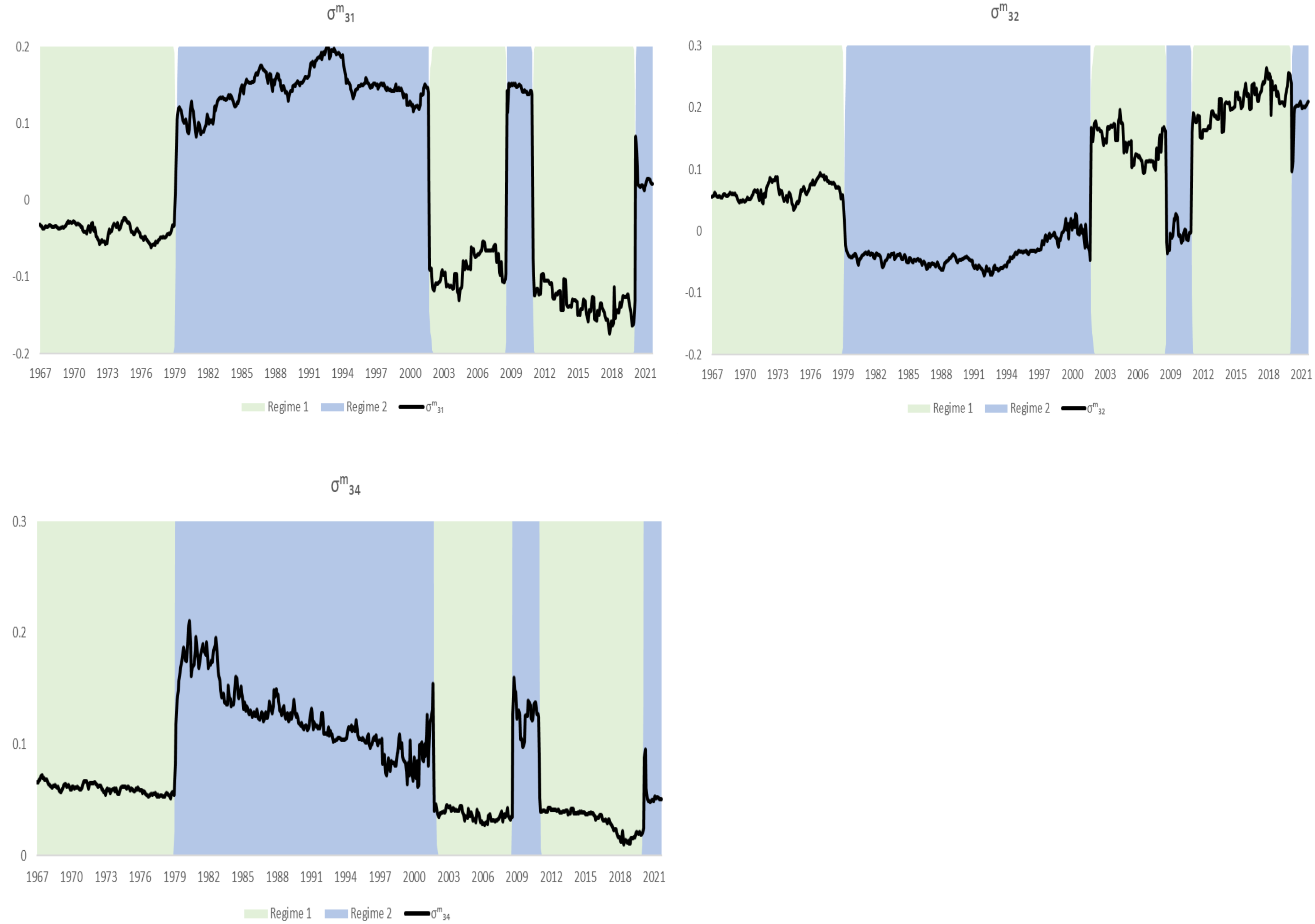
Note: 1 = Consumption, 2 = Leisure, 3 = Monetary aggregate, 4 = Aggregate of non-money monetary assets

Appendix Figure A14. Morishima elasticities of substitution for leisure based on the Markov-switching NQ demand system and Divisia M3



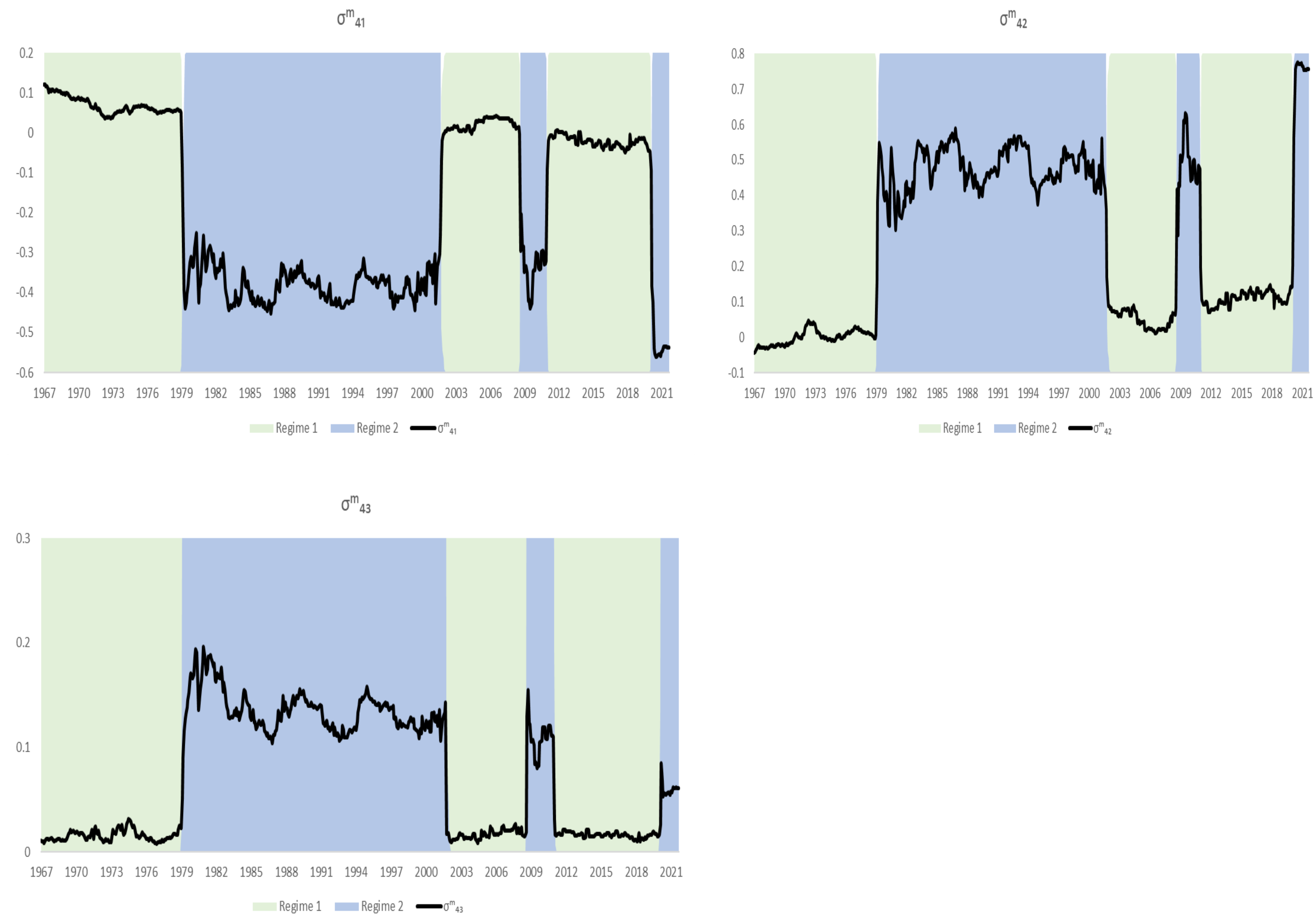
Note: 1 = Consumption, 2 = Leisure, 3 = Monetary aggregate, 4 = Aggregate of non-money monetary assets

Appendix Figure A15. Morishima elasticities of substitution for money based on the Markov-switching NQ demand system and Divisia M3



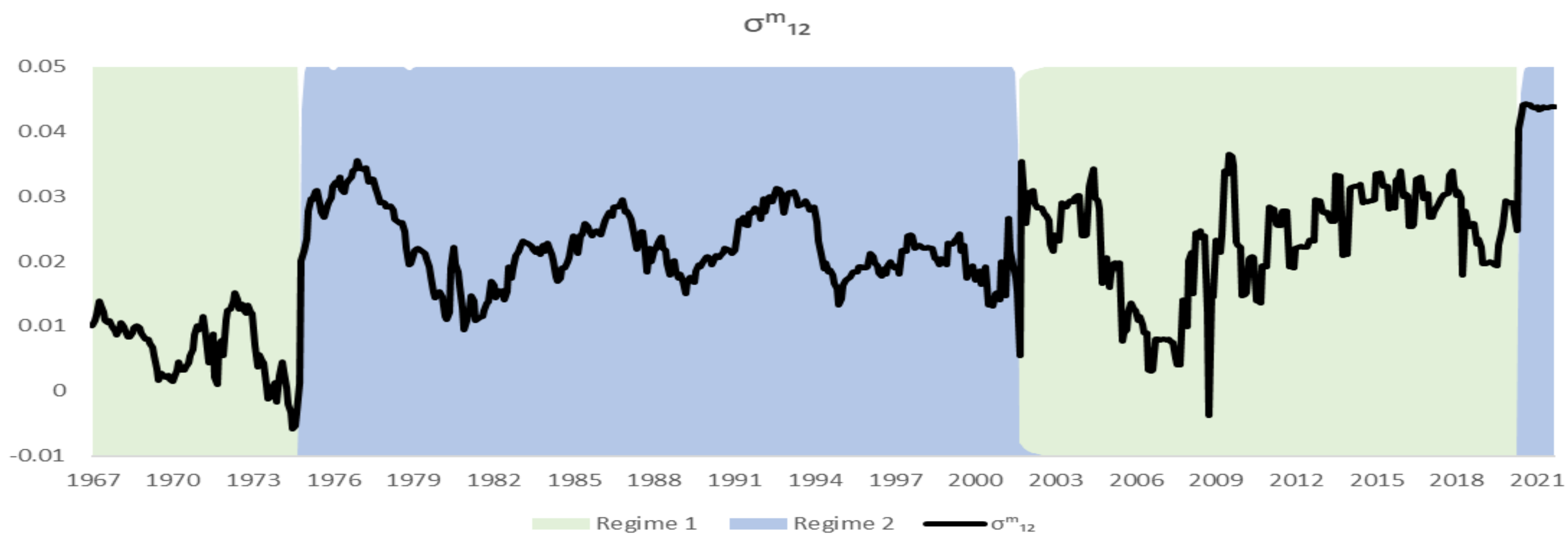
Note: 1 = Consumption, 2 = Leisure, 3 = Monetary aggregate, 4 = Aggregate of non-money monetary assets

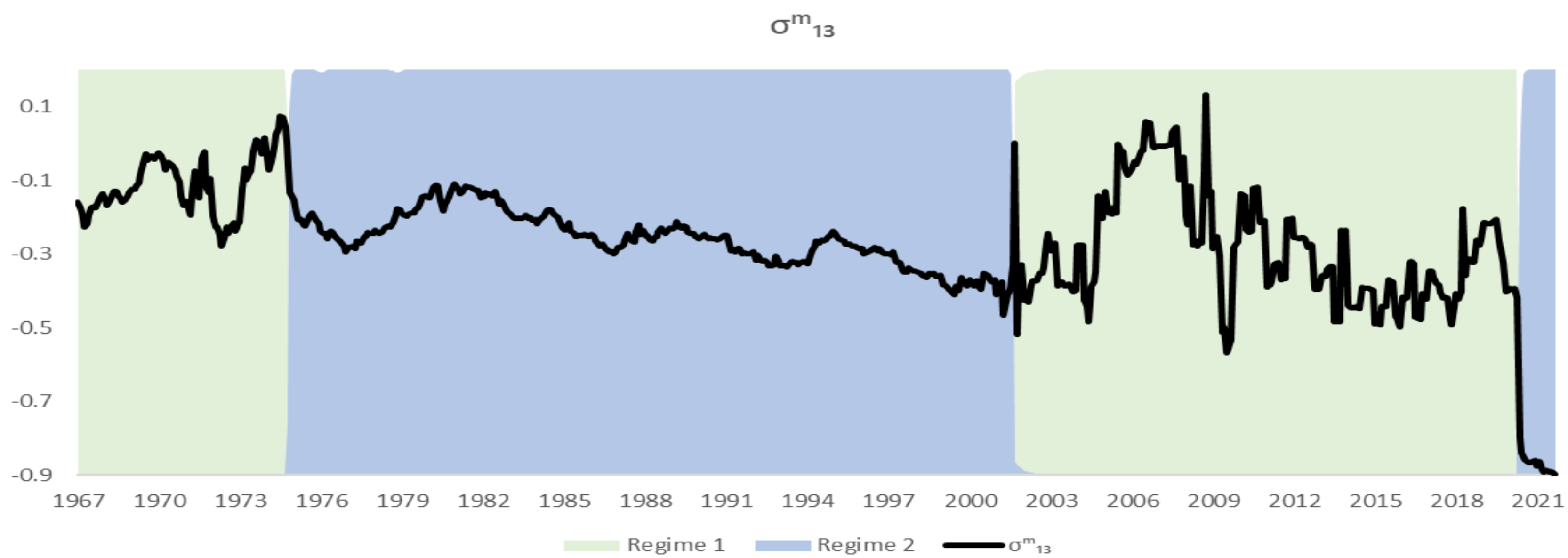
Appendix Figure A16. Morishima elasticities of substitution for other monetary assets based on the Markov-switching NQ demand system and Divisia M3



Note: 1 = Consumption, 2 = Leisure, 3 = Monetary aggregate, 4 = Aggregate of non-money monetary assets

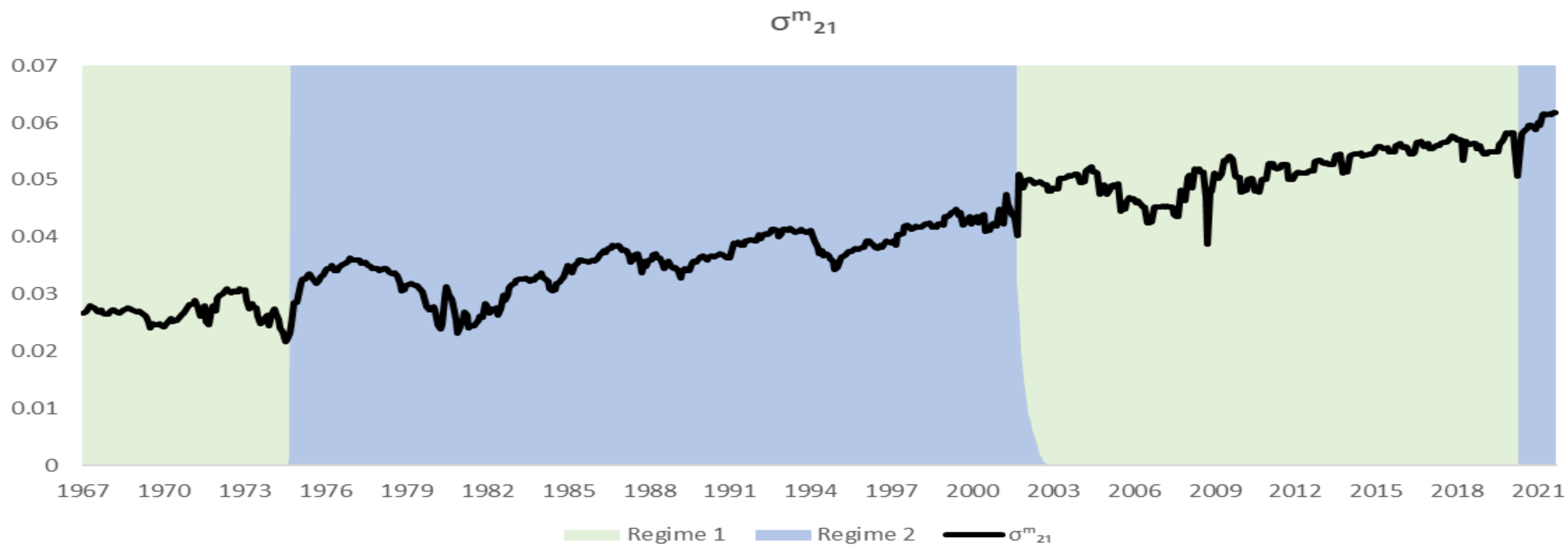
Appendix Figure A17. Morishima elasticities of substitution for consumption based on the Markov-switching NQ demand system and Divisia M4

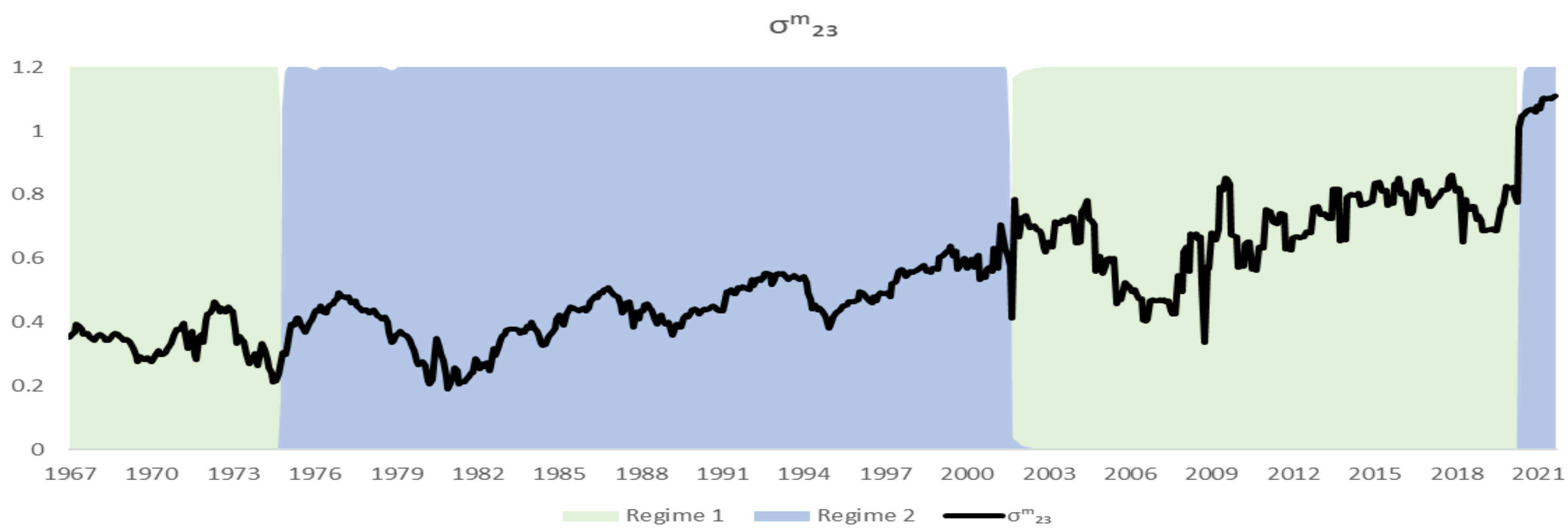




Note: 1 = Consumption, 2 = Leisure, 3 = Monetary aggregate,

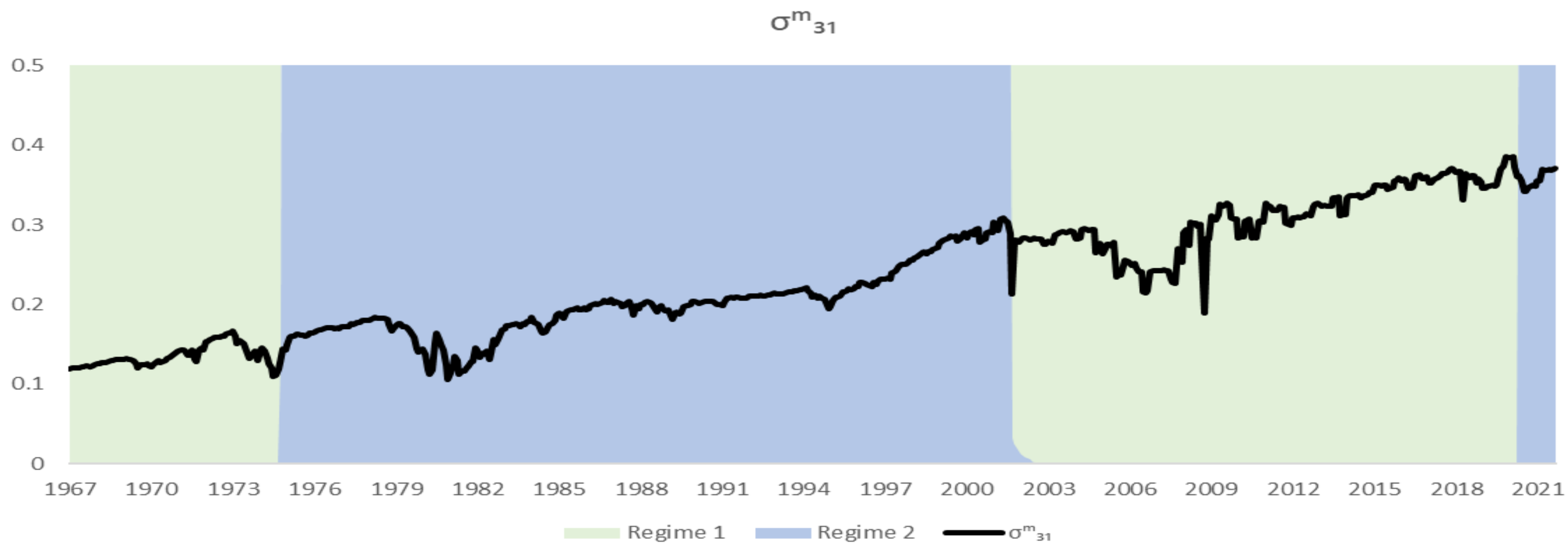
Appendix Figure A18. Morishima elasticities of substitution for leisure based on the Markov-switching NQ demand system and Divisia M4

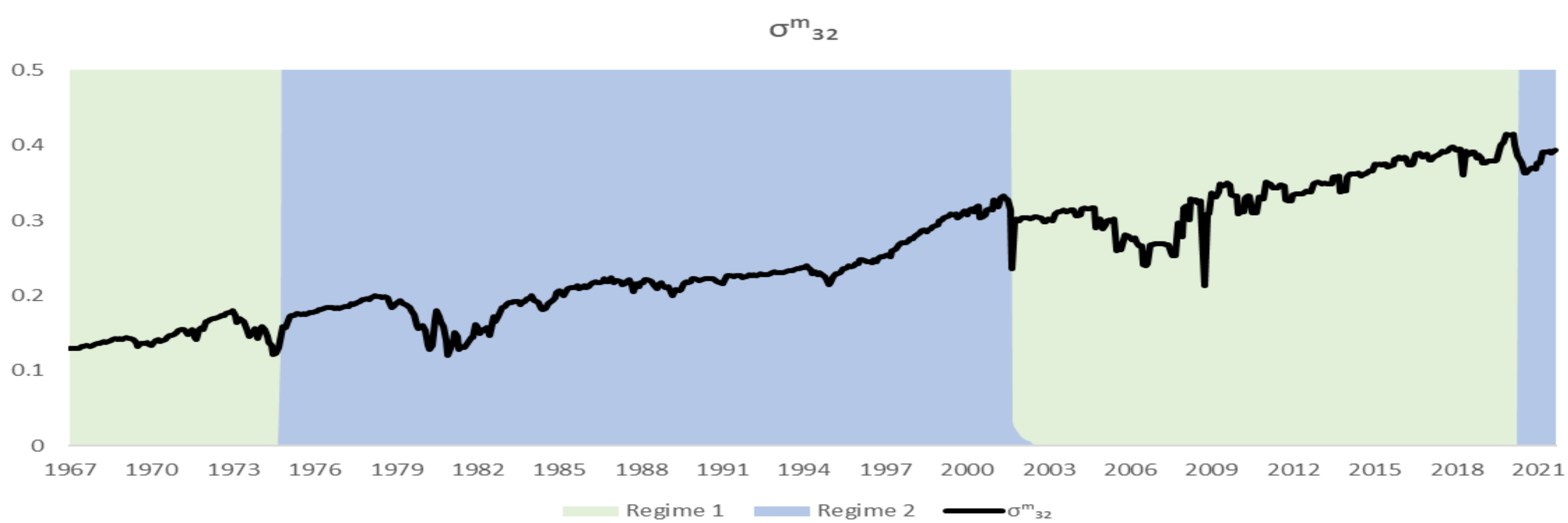




Note: 1 = Consumption, 2 = Leisure, 3 = Monetary aggregate

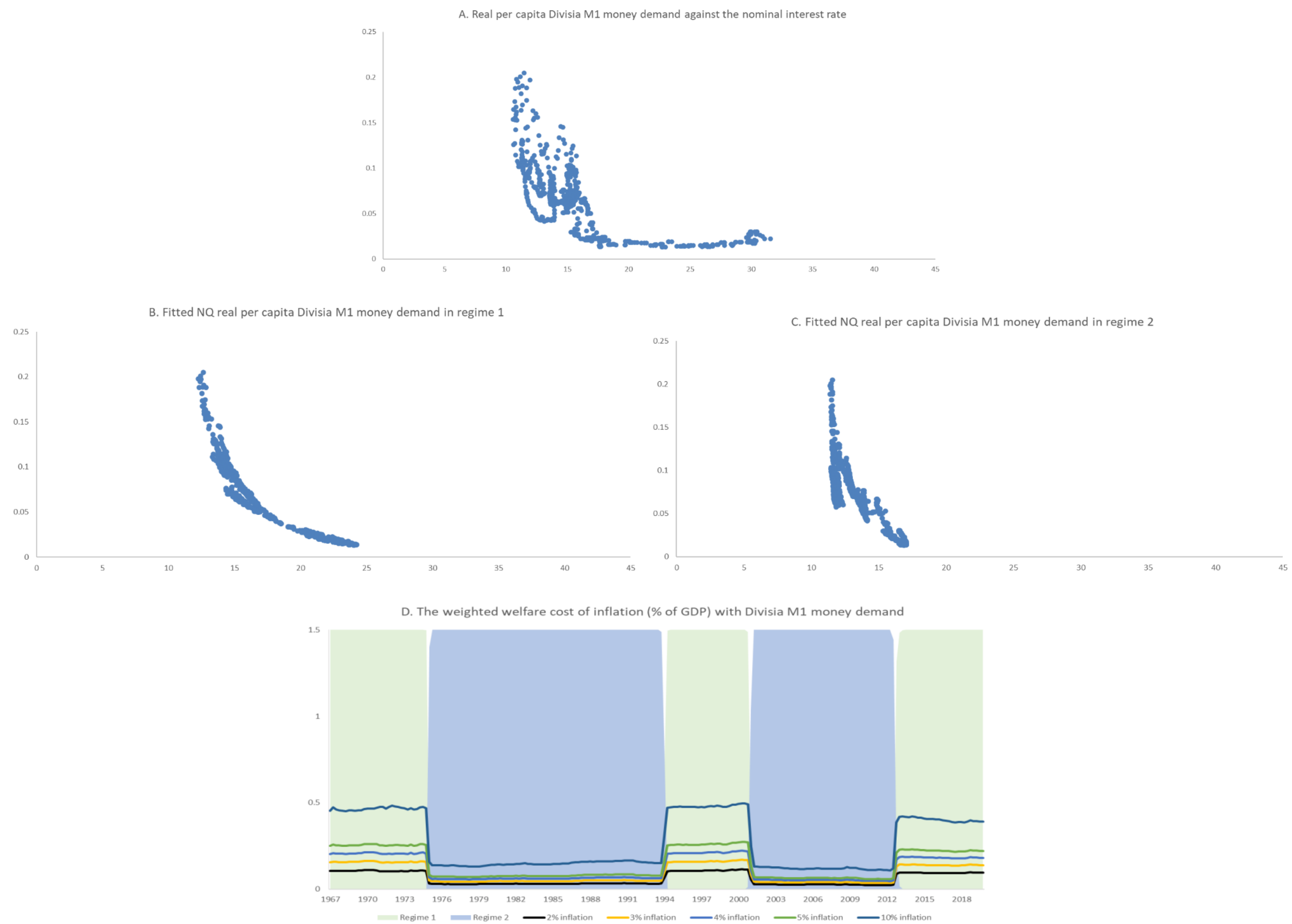
Appendix Figure A19. Morishima elasticities of substitution for money based on the Markov-switching NQ demand system and Divisia M4





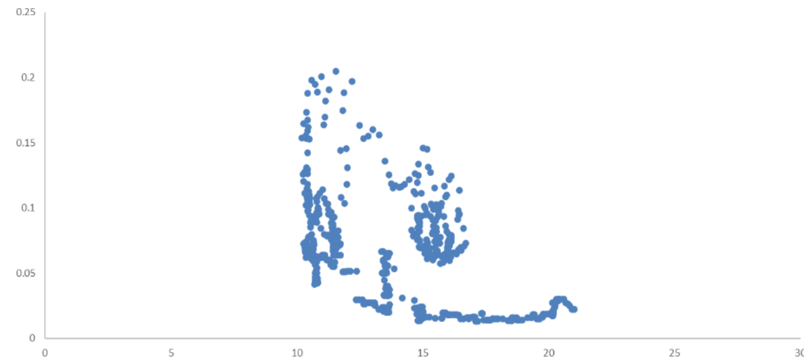
Note: 1 = Consumption, 2 = Leisure, 3 = Monetary aggregate

Appendix Figure A20. The Demand for Divisia M1 (in real per capita terms) with pre-COVID sample

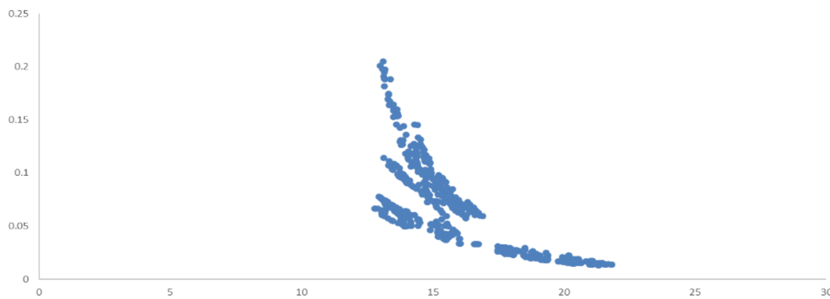


Appendix Figure 21. The Demand for Divisia M2 (in real per capita terms) with pre-COVID sample

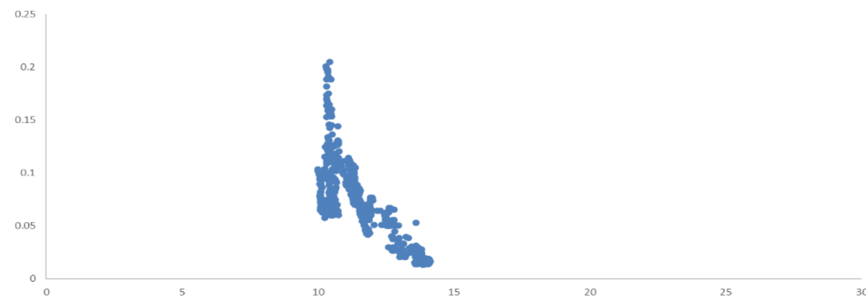
A. Real per capita Divisia M2 money demand against the nominal interest rate



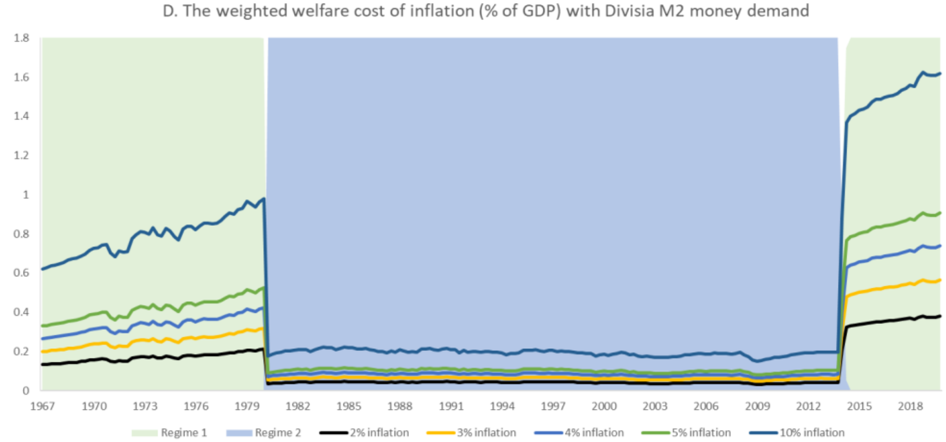
B. Fitted NQ real per capita Divisia M2 money demand in regime 1



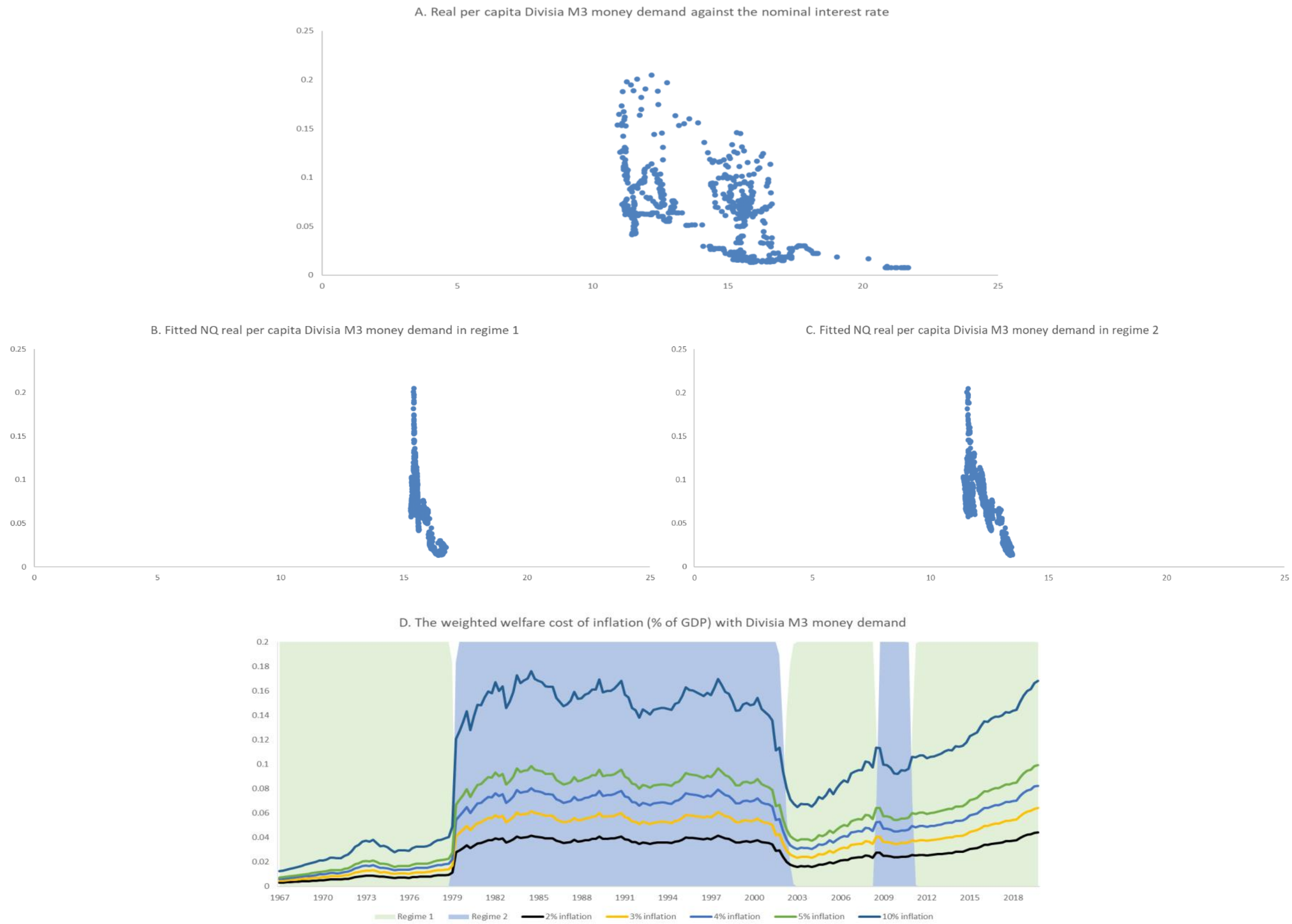
C. Fitted NQ real per capita Divisia M2 money demand in regime 2



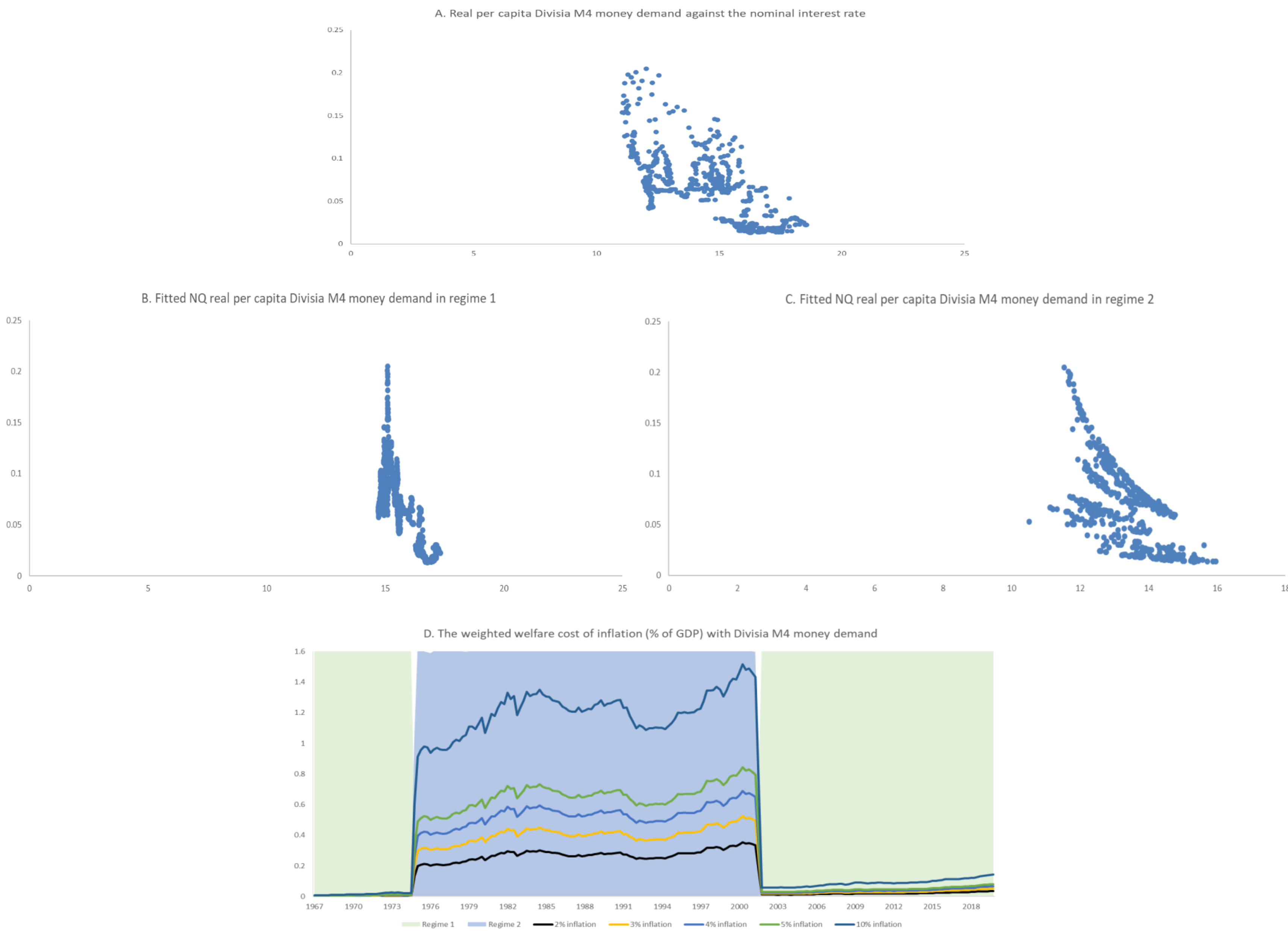
D. The weighted welfare cost of inflation (% of GDP) with Divisia M2 money demand



Appendix Figure 22. The Demand for Divisia M3 (in real per capita terms) with pre-COVID sample



Appendix Figure 23. The Demand for Divisia M4 (in real per capita terms) with pre-COVID sample



Appendix Table A8. Regime-dependent welfare costs of inflation

	Inflation				
	2%	3%	4%	5%	10%
			Divisia M1		
Regime 1	0.1027	0.1523	0.2001	0.2461	0.4487
Regime 2	0.0289	0.0434	0.0578	0.0718	0.1366
			Divisia M2		
Regime 1	0.2324	0.3462	0.4564	0.5622	1.0260
Regime 2	0.0413	0.0619	0.0822	0.1019	0.1936
			Divisia M3		
Regime 1	0.0177	0.0229	0.0343	0.0416	0.0724
Regime 2	0.0363	0.0533	0.0694	0.0845	0.1486
			Divisia M4		
Regime 1	0.0147	0.0219	0.0289	0.0357	0.0671
Regime 2	0.2730	0.4065	0.5360	0.6608	1.2123

Note: Sample, 1967q1-2019q4. Numbers are mean values (in %).