Unsolved Problems in Monetary Aggregation and Transmission

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The New Divisia Monetary Index: The Joint Services of Money and Credit

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Introduction

Credit card transactions $\subset$ monetary supply components?

Introduction

Accounting versus microeconomic aggregation theory.

Introduction

Credit cards have never been included in measures of the money supply. The reason is accounting conventions, which do not permit adding liabilities, such as credit card balances, to assets, such as money. However, economic aggregation theory and index number theory measure service flows and are based on **microeconomic theory, not accounting**.

Notes:

Credit cards include Visa cards, Mastercards, Discover Cards, and those American Express account cards providing a line of credit. Our data are from the annual reports of those four sources. Not all American Express cards are "credit cards." Some American Express account cards are "charge cards." **We do not include charge cards, store cards, or debit cards.**

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$\mathbf{m}_s$ = per capita real balances of monetary assets during period s.

c_{js} = per capita real expenditure with credit card type j for transactions during period s.

z_{js} = per capita rotating real balances in credit card type j during period s from transactions in previous periods.

$y_{js} = c_{js} + z_{js}$ = per capita total balances in credit type j during period s.

r_{is} = expected nominal holding period yield on monetary asset i during period s.

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e_{js} = expected interest rate on c_{js}.
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The Federal Reserve reports two credit card interest rates. One is the interest rate averaged over those accounts charged interest. That is the interest rate on z_{js}. The other is the interest rate averaged over all credit card accounts, including those accounts not being charged interest, since paid off before the end of the month. We use that measure as e_{js}, which is less than the interest rate on z_{js}. **Our model is of the representative consumer, averaged over all consumers, including those not paying interest on their credit cards.**

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Intertemporal Allocation

$$\max u_t = u_t(\mathbf{m}_t,\ldots,\mathbf{m}_{t+T};\mathbf{c}_t,\ldots,\mathbf{c}_{t+T};\mathbf{x}_t,\ldots,\mathbf{x}_{t+T};A_{t+T})$$
$$= U_t(v(\mathbf{m}_t,\mathbf{c}_t),v_{t+1}(\mathbf{m}_{t+1},\mathbf{c}_{t+1}),\ldots,v_{t+T}(\mathbf{m}_{t+T},\mathbf{c}_{t+T});$$
$$V(\mathbf{x}_t),V_{t+1}(\mathbf{x}_{t+1}),\ldots,V_{t+T}(\mathbf{x}_{t+T});A_{t+T})$$

subject to

$$\mathbf{p}'_s\mathbf{x}_s = \omega_s L_s + \sum_{i=1}^{n}[(1+r_{i,s-1})p^*_{s-1}m_{i,s-1} - p^*_s m_{is}]$$
$$+\sum_{j=1}^{k}[p^*_s c_{js} - (1+e_{j,s-1})p^*_{s-1}c_{j,s-1}]$$
$$+\sum_{j=1}^{k}[p^*_s z_{js} - (1+\bar{e}_{j,s-1})p^*_{s-1}z_{j,s-1}]$$
$$+[(1+R_{s-1})p^*_{s-1}A_{s-1} - p^*_s A_s].$$

Notes:

Carried forward rotating balances are not included in the current period weakly separable block, $v(\mathbf{m}_t, \mathbf{c}_t)$, since those balances were used for transactions in prior periods. To include them in that block would produce double counting of those services. But those rotating balances do appear in the consumer's budget constraints as well as in the weakly separable block during the period used for transactions. In the jargon of the credit card industry, credit card balances used for current period transactions services are called "**volumes**." We aggregate over those volumes.

Notes:

The benchmark rate, R_t, is the rate of return on the classical, owned pure capital asset, providing no services other than its financial rate of return. Since owned and can be sold to recover its cost, **that asset is 100% secured**. However, consumer credit card balances are not assets.

Notes:

Credit card interest rates are much higher than interest rates paid on monetary assets, since credit card accounts are liabilities and are unsecured. Credit cards are also subject to fraud risk. **Over 80% of credit card accounts are charged interest at very high rates.** According to the Federal Reserve's data, the average rate of return on credit card accounts, e_{js}, including those not charged interest, far exceeds our benchmark rate of return, R_t.

Intertemporal Allocation

Let

$$\rho = \begin{cases} 1, & \text{if } s = t, \\ \prod_{u=t}^{s-1}(1 + R_u), & \text{if } t + 1 \leq s \leq t + T. \end{cases}$$

Intertemporal Allocation

The nominal user cost of monetary asset holding m_{is} is

$$\pi_i = \frac{p_s^*}{\rho_s} - \frac{p_s^*(1 + r_{is})}{\rho_{s+1}}$$
$$= p_t^* \frac{R_t - r_{it}}{1 + R_t}.$$

Intertemporal Allocation

Likewise, the nominal user cost of credit card service c_{js} is

$$\tilde{\pi}_j = \frac{p_s^*(1+e_{js})}{\rho_{s+1}} - \frac{p_s^*}{\rho_s}$$
$$= p_t^* \frac{e_{jt} - R_t}{1 + R_t}.$$

Aggregation Theory

The exact quantity aggregate is the level of the indirect utility produced by the utility maximization problem:

$$
\begin{aligned}
\mathscr{M} &= \max\{v(\mathbf{m}_t,\mathbf{c}_t) : \boldsymbol{\pi}'_t\boldsymbol{m}_t + \tilde{\boldsymbol{\pi}}'_t\mathbf{c}_t = \mathscr{I}_t\} \\
&= \max\{v(\mathbf{m}_t,\mathbf{c}_t) : \boldsymbol{\pi}^{*\prime}_t\mathbf{m}_t + \tilde{\boldsymbol{\pi}}^{*\prime}_t\mathbf{c}_t = \mathscr{I}^*_t\} \\
&= v(\mathbf{m}_t,\mathbf{c}_t) \\
&= \mathscr{M}(\mathbf{m}_t,\mathbf{c}_t) \\
&= \text{Augmented Monetary Aggregate.}
\end{aligned}
$$

Aggregation Theory

An exact dual pair of price and quantity aggregates satisfies Fisher's factor reversal test:

$$\Pi(\boldsymbol{\pi}_t, \tilde{\boldsymbol{\pi}}_t) = \frac{\mathscr{I}_t}{\mathscr{M}_t}$$

Aggregation Theory

It can be proved that

$$\Pi(\boldsymbol{\pi}_t, \tilde{\boldsymbol{\pi}}_t) = E(1, \boldsymbol{\pi}_t, \tilde{\boldsymbol{\pi}}_t) = \min_{\{\mathbf{m}_t, \mathbf{c}_t\}} \{\boldsymbol{\pi}_t' \mathbf{m}_t + \tilde{\boldsymbol{\pi}}_t' \mathbf{c}_t : \nu(\mathbf{m}_t, \mathbf{c}_t) = 1\}.$$

So

$$\Pi(\boldsymbol{\pi}_t, \tilde{\boldsymbol{\pi}}_t) = \frac{\mathscr{I}_t}{\mathscr{M}_t} = E(1, \boldsymbol{\pi}_t, \tilde{\boldsymbol{\pi}}_t).$$

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So

$$\Pi(\boldsymbol{\pi}_t,\tilde{\boldsymbol{\pi}}_t) = \frac{\mathscr{I}_t}{\mathscr{M}_t} = E(1,\boldsymbol{\pi}_t,\tilde{\boldsymbol{\pi}}_t).$$

The Resulting Quantity Aggregator Functions

In summary, we have

$$M_t = \max\{g_1(\mathbf{m}_t) : \boldsymbol{\pi}_t^{*\prime}\mathbf{m}_t = \Pi_m^* M_t\}$$

and

$$C_t = \max\{g_2(\mathbf{c}_t) : \tilde{\boldsymbol{\pi}}_t^{*\prime}\mathbf{c}_t = \Pi_c^* C_t\}.$$

Thus, the optimal values of the monetary and credit card quantity aggregates are related to the joint aggregate in the following manner:

$$\mathscr{M}_t = \tilde{v}(M_t, C_t).$$

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The Divisia Index

Discrete Time Approximations to the Divisia Index: the Törnqvist-Theil Approximation.

$$\log \mathscr{M}(\mathbf{m}_t^a) - \log \mathscr{M}(\mathbf{m}_{t-1}^a)$$

$$= \sum_{i=1}^{n} \bar{\omega}_{it}(\log m_{it} - \log m_{i,t-1}) + \sum_{i=1}^{k} \bar{\tilde{\omega}}_{it}(\log c_{it} - \log c_{i,t-1}),$$

where $\bar{\omega}_{it} = (\omega_{it} + \omega_{i,t-1})/2$, $\bar{\tilde{\omega}}_{it} = (\tilde{\omega}_{it} + \tilde{\omega}_{i,t-1})/2$, $\omega_{it} = \pi_{it} m_{it}/(\boldsymbol{\pi}_t' \mathbf{m}_t + \tilde{\boldsymbol{\pi}}_t' \mathbf{c}_t)$, and $\tilde{\omega}_{it} = \tilde{\pi}_{it} c_{it}/(\boldsymbol{\pi}_t' \mathbf{m}_t + \tilde{\boldsymbol{\pi}}_t' \mathbf{c}_t)$.

The Nowcasting Model

We use the optimal indicators found by Barnett, Chauvet, and Leiva-Leon (2016)[1], but with credit card transactions volumes included in the indicator set along with the Divisia monetary aggregates. Accordingly, we use Nominal GDP, $y_{1,t}$, Industrial Production, $y_{2,t}$, Consumer Price Index, $y_{3,t}$, a Divisia Monetary Aggregate, $y_{4,t}$, and Credit Card Expenditure Volume, $y_{5,t}$, to estimate the following Mixed Frequency Dynamic Factor model:

[1] Barnett, W. A., M. Chauvet, and D. Leiva-Leon (2016). "Real-Time Nowcasting of Nominal GDP Under Structural Break," *Journal of Econometrics*, 191(2), 312-324.

Application Specific Aggregation

Indicator-Specific Aggregation

The aggregation theoretic augmented monetary aggregate,

$$\mathscr{M}_t = \tilde{v}(M_t, C_t),$$

can be considered for many possible uses. For example, the aggregate can be considered as an intermediate target for policy, as a Taylor rule target, as a long run anchor for monetary policy, or as the dependent variable in a demand for monetary services equation. The assumption of weak separability in tastes is needed for existence of such a micro-founded aggregate. But an alternative augmented aggregate, optimized and specific for use as an indicator, can exist without that weak separability assumption.

Application Specific Aggregation

Nowcasting Optimized Monetary Indicator

Nowcasting of nominal GDP can independently use both components of the two dimensional vector, $\mathbf{M}_t = (M_t, C_t)$, without the need to assume the existence of the aggregator function, $\tilde{v}$. The completed nowcasting procedure would itself result in an optimal scalar function,

$$\mathscr{M}_t^* = g(\mathbf{M}_t),$$

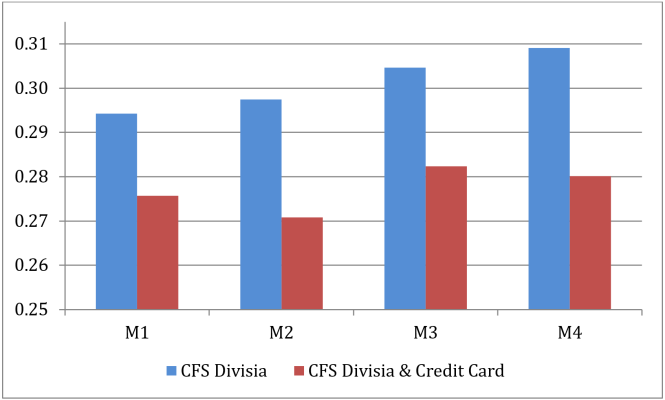
of the two elements of (M_t, C_t). Existence of the optimal indicator aggregator function, g, is produced by its **weak separability within the solution nowcasting equation**.

Application Specific Aggregation

The following slides compare the regular Divisia monetary aggregates, $M_t = M(\mathbf{m}_t)$, with the application-specific augmented aggregator functions, $\mathscr{M}_t^* = g(M_t, C_t)$, as indicators of monthly nominal GDP.

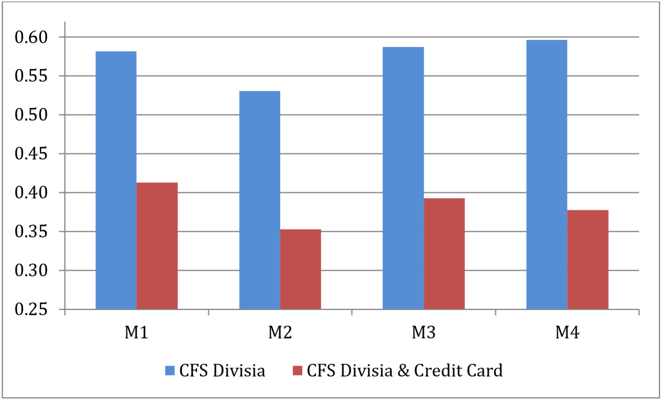
Real Time Analysis

Figure 1. Mean Square Error Comparison (Full sample)



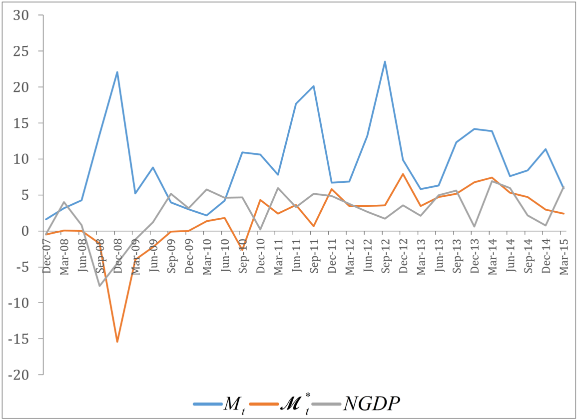
Real Time Analysis

Figure 2. Mean Square Error Comparison (Great Recession)



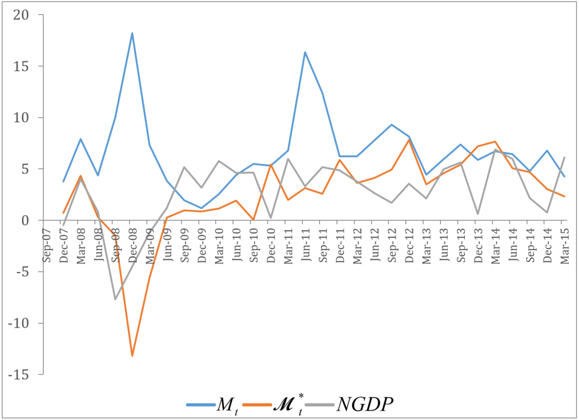
Real Time Analysis

Figure 3. M1 Average Quarterly Growth Rates (2007Q4 - 2015Q1)



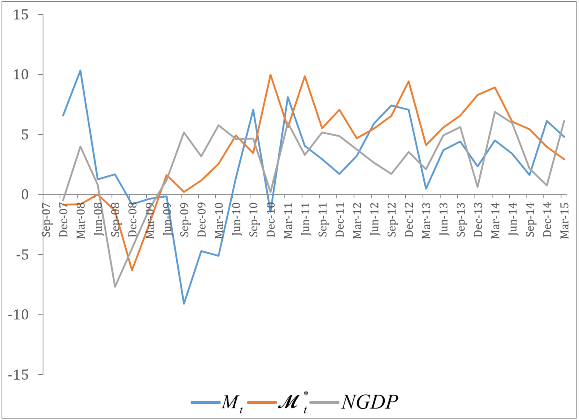
Real Time Analysis

Figure 4. M2 Average Quarterly Growth Rates (2007Q4 - 2015Q1)



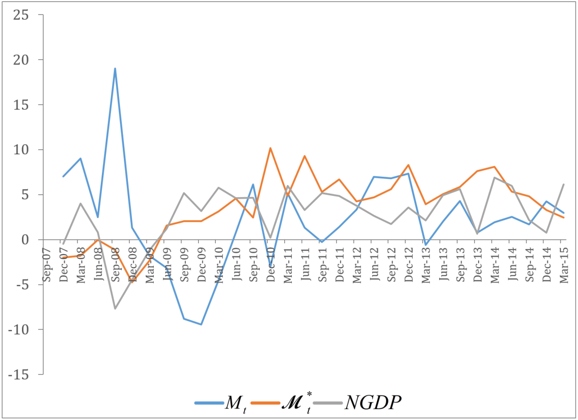
Real Time Analysis

Figure 5. M3 Average Quarterly Growth Rates (2007Q4 - 2015Q1)



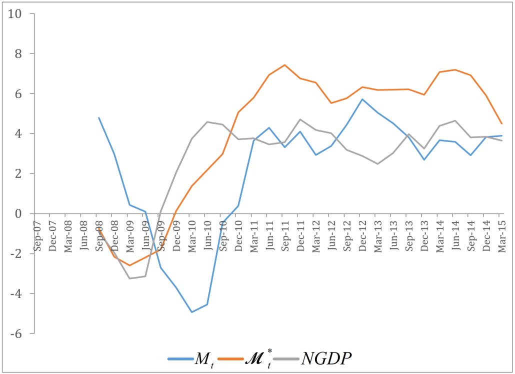
Real Time Analysis

Figure 6. M4 Average Quarterly Growth Rates (2007Q4 - 2015Q1)



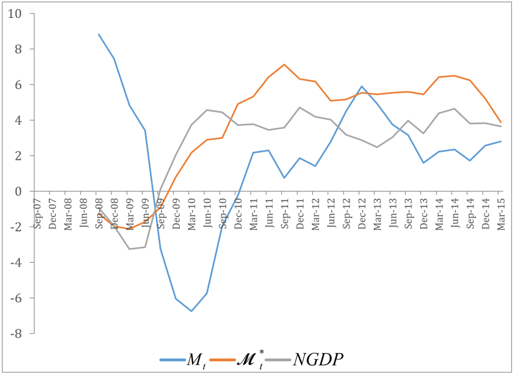
Real Time Analysis

Figure 9. M3 Quarterly Year-over-Year Growth Rates (2007Q4 - 2015Q1)



Real Time Analysis

Figure 10. M4 Quarterly Year-over-Year Growth Rates (2007Q4-2015Q1)



Risk

- William A. Barnett and Liting Su, “Risk Adjustment of the Credit-Card Augmented Divisia Monetary Augmented Divisia Monetary Aggregates.” In Giovanni De Bartolomeo, Daniela Federici, and Enrico Saltari (eds.), Macroeconomic Advances in Honor of Clifford Wymer, special issue of *Macroeconomic Dynamics*, forthcoming.
- CCAPM “equity premium puzzle.”
- **Aggregation over consumers faced with different user cost prices. Existence of representative consumer.**
  - Aggregation can smooth consumption.
  - Aggregation can increase volatility of credit card interest rate.
- Implicit interest rate on credit card transactions.

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Public Data Availability

In the near future, the aggregation-theoretic augmented Divisia monetary aggregates, $\mathscr{M}_t$, and the indicator-optimized augmented Divisia monetary aggregates, $\mathscr{M}_t^*$, will begin to be made available to the public by the Center for Financial Stability (CFS) in New York City through monthly releases, and by Bloomberg to its terminal users.

Public Data Availability

http://www.centerforfinancialstability.org/

Financial Firm Production of Monetary and Credit Card Services: An Aggregation Theoretic Approach

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Introduction

- A monetary-production model of financial firms is employed to investigate supply-side monetary aggregation, augmented to include credit card transaction services.
- Financial firms are conceived to produce monetary and credit card transaction services as outputs through financial intermediation.
- In this paper, we derive theory needed to measure the supply of the joint services of credit cards and money.
- The resulting model can be used to investigate the **transmission mechanism of monetary policy**.

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Introduction

- Barnett's (1987) monetary aggregation-theoretic approach.
- Hancock's (1991) approach.
- Extension: to include production of credit card transactions services.
- Financial firms are modeled as maximizing the discounted present value of variable profits, subject to given technology, while producing monetary assets and credit card services through financial intermediation.
- With the derivation of user-cost prices for monetary assets and credit card transaction services, the monetary production model can be transformed into the conventional neoclassical model of production by multiproduct firms.

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The Model

First, we define the variables that are used in the financial intermediary's decision problem:

R_t = yield on the benchmark asset;

$\boldsymbol{\mu}_t$ = real balances of monetary asset accounts serviced by the financial intermediary;

$\boldsymbol{\tau}_t$ = vector of real expenditure "volumes," τ_{jt}, with credit card type j for transactions during period t;

$\mathbf{e}_t$ = vector of expected interest rates on $\boldsymbol{\tau}_t$;

$\boldsymbol{\zeta}_t$ = vector of rotating real balances, ζ_{jt}, in credit card type j during period t from transactions in previous periods;

$\bar{\mathbf{e}}_t$ = vector of interest rates on $\boldsymbol{\zeta}_t$.

c_t = real balances of excess reserves held by the intermediary during period t;

$\mathbf{L}_t$ = vector of labor quantities;

$\mathbf{z}_t$ = quantities of other factors of production;

$\mathbf{q}_t$ = prices of the factors, $\mathbf{z}_t$;

$\mathbf{k}_t$ = reserve requirements, where k_{it} is the reserve requirement applicable to μ_{it} and $0 \leq k_{it} \leq 1$ for all i;

R_t^d = Federal Reserve discount rate;

$\overline{R}_t = \min\{R_t, R_t^d\}$;

$\boldsymbol{\rho}_t$ = vector of yields paid by the firm on $\boldsymbol{\mu}_t$.

The firm's efficient production technology is defined by the transformation function $F(\boldsymbol{\mu}_t, \boldsymbol{\tau}_t, \mathbf{z}_t, \mathbf{L}_t, c_t; \mathbf{k}_t) = 0$, assumed to be strictly quasiconvex in $(\boldsymbol{\mu}_t, \boldsymbol{\tau}_t, \mathbf{z}_t, \mathbf{L}_t, c_t)$, strictly increasing in outputs $(\boldsymbol{\mu}_t, \boldsymbol{\tau}_t)$ and strictly decreasing in inputs $(\mathbf{z}_t, \mathbf{L}_t, c_t)$.

Since the intermediary's servicing of credit card transactions are during the current period, the firm's production technology includes $\boldsymbol{\tau}_t$ but does not include $\boldsymbol{\zeta}_t$. **The value added in servicing transactions occurs during the period when the credit cards are used for transactions.** The service provided is deferred payment, not possible with cash, demand deposits, or debit cards.

- Rotating credit card balances are relevant to the credit channel of monetary policy, to be incorporated in a later extension of this research.

Since the intermediary's servicing of credit card transactions are during the current period, the firm's production technology includes $\boldsymbol{\tau}_t$ but does not include $\boldsymbol{\zeta}_t$. **The value added in servicing transactions occurs during the period when the credit cards are used for transactions.** The service provided is deferred payment, not possible with cash, demand deposits, or debit cards.

- Rotating credit card balances are relevant to the credit channel of monetary policy, to be incorporated in a later extension of this research.

We assume that required reserves are never borrowed from the Federal Reserve, but could be borrowed in the federal funds market. Excess reserves can be borrowed from either source.

In this initial model, we assume that the Federal Reserve does not pay interest on reserves, as has been the case during most of its history. Since we are assuming the existence of only one kind of primary market loan yielding R_t, it follows that the federal funds rate must always equal R_t .

In subsequent research we will introduce loan servicing costs to permit **value added in lending**. The extended model will be relevant to the credit channel of policy, through a different weakly separable output block in technology. Serviced loans will yield a higher rate of return than loans made by purchasing bonds, which are primary securities, producing no value added in banking.

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- If $R_t^d < R_t$,
  then all excess reserves will be borrowed from the Federal Reserve and there are no free reserves.

- If $R_t^d > R_t$,
  then there is no borrowing from the Federal Reserve and free reserves equal excess reserves.

- If $R_t \leq R_t^d$, then $\overline{R}_t = R_t$, and revenue from loans is

$$\sum_i (\mu_{it} p_t^* - \sum_i k_{it} \mu_{it} p_t{}^* - \sum_j p_t^* \tau_{jt} - \sum_j p_t^* \zeta_{jt} - c_t p_t{}^* - \mathbf{q}_t' \mathbf{z}_t) R_t + \sum_j e_{jt} p_t^* \tau_{jt} + \sum_j \overline{e}_{jt} p_t^* \zeta_{jt}.$$

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Hence, in either case, **revenue from loans** is

$$[\sum_i (1-k_{it})\mu_{it} p_t^* - c_t p_t^* - \mathbf{q}_t' \mathbf{z}_t - \sum_j p_t^* \tau_{jt} - \sum_j p_t^* \zeta_{jt}] R_t$$
$$+ c_t {p_t}^* (R_t - \overline{R}_t) + \sum_j e_{jt} p_t^* \tau_{jt} + \sum_j \overline{e}_{jt} p_t^* \zeta_{jt}.$$

Variable cost, which must be paid out of revenue, is

$$\sum_i \mu_{it} p_t^* \rho_{it} + \mathbf{q}_t' \mathbf{z}_t + \mathbf{w}_t' \mathbf{L}_t.$$

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At the end of period t , profit received is acquired by subtracting variable cost from revenue from loans. Dividing by $1 + R_t$ to discount profits to the beginning of period t, we get the present value of period t profits to be

$$P(\boldsymbol{\mu}_t, \boldsymbol{\tau}_t, \boldsymbol{\zeta}_t, \mathbf{z}_t, \mathbf{L}_t, c_t; p_t^*, \mathbf{q}_t, R_t, {R_t}^d, \mathbf{e}_t, \bar{\mathbf{e}}_t, \boldsymbol{\rho}_t, \mathbf{w}_t, \mathbf{k}_t)$$
$$= \boldsymbol{\mu}_t' \boldsymbol{\gamma}_t + \boldsymbol{\tau}_t' \tilde{\boldsymbol{\pi}}_{\mathbf{t}} + \boldsymbol{\zeta}_t' \boldsymbol{\sigma}_t - \mathbf{q}_t' \mathbf{z}_t - \mathbf{L}_t' \mathbf{w}_t / (1 + R_t) - \gamma_{0t} c_t,$$

- where the vector $\boldsymbol{\gamma}_t$ is defined such that the nominal user cost price for produced monetary asset μ_{it} is

$$\gamma_{it} = p_t^* \frac{(1 - k_{it})R_{it} - \rho_{it}}{1 + R_t},$$

- the vector $\tilde{\boldsymbol{\pi}}_{\mathbf{t}}$ is defined such that the nominal user cost price for produced credit card service τ_{jt} is

$$\tilde{\pi}_{jt} = p_t^* \frac{e_{jt} - R_t}{1 + R_t},$$

- the vector $\boldsymbol{\sigma}_t$ is defined such that the nominal user cost price for carried forward rotating credit card ζ_{jt} is

$$\sigma_{jt} = p_t^* \frac{\bar{e}_{jt} - R_t}{1 + R_t},$$

- and the nominal user cost price of excess reserves, c_t , is

$$\gamma_{ot} = p_t^* \frac{\overline{R}_t}{1 + R_t}.$$

- The corresponding real user costs are $\frac{\boldsymbol{\gamma}_t}{p_t^*}$, $\frac{\widetilde{\boldsymbol{\pi}}_t}{p_t^*}$, $\frac{\boldsymbol{\sigma}_t}{p_t^*}$, and $\frac{\gamma_{ot}}{p_t^*}$.

If we write the vector of all **variable factor quantities** as

$$\boldsymbol{\alpha}_t = (\mathbf{z}_t', \mathbf{L}_t', c_t)'$$

and the vector of corresponding **factor prices** as

$$\boldsymbol{\beta}_t = (\mathbf{q}_t', \mathbf{w}_t'/(1+R_t), \gamma_{ot})',$$

it becomes evident that **profits** take the conventional form

$$\boldsymbol{\mu}_t'\boldsymbol{\gamma}_t + \boldsymbol{\tau}_t'\tilde{\boldsymbol{\pi}}_t + \boldsymbol{\zeta}_t'\boldsymbol{\sigma}_t - \boldsymbol{\alpha}_t'\boldsymbol{\beta}_t.$$

But since the financial firm's decision is conditional upon consumer choice of $\boldsymbol{\zeta}_t$, variable profits can be written as

$$P_t = \boldsymbol{\mu}_t'\boldsymbol{\gamma}_t + \boldsymbol{\tau}_t'\tilde{\boldsymbol{\pi}}_t - \boldsymbol{\alpha}_t'\boldsymbol{\beta}_t,$$

and the firm's variable profit maximization problem takes the conventional form of selecting $(\boldsymbol{\mu}_t, \boldsymbol{\tau}_t, \mathbf{z}_t, \mathbf{L}_t, c_t) \in S(\mathbf{k}_t)$ to maximize P_t.

Hence the existing literature on output aggregation for multiproduct firms becomes immediately applicable to aggregation over the produced monetary services $(\boldsymbol{\mu}_t, \boldsymbol{\tau}_t)$ and to measuring value added and technological change in financial intermediation.

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Separability of Technology

Following Barnett (1987), we assume there exist functions f and H such that

$$F(\boldsymbol{\mu}_t, \boldsymbol{\tau}_t, \mathbf{z}_t, \mathbf{L}_t, c_t; \mathbf{k}_t) = H(f(\boldsymbol{\mu}_t, \boldsymbol{\tau}_t; \mathbf{k}_t), \mathbf{z}_t, \mathbf{L}_t, c_t).$$

Under the usual neoclassical assumptions on technology, there will exist a function g such that

$$\boxed{f(\boldsymbol{\mu}_t, \boldsymbol{\tau}_t; \mathbf{k}_t) = g(\mathbf{z}_t, \mathbf{L}_t, c_t)}$$

is the solution for $f(\boldsymbol{\mu}_t, \boldsymbol{\tau}_t; \mathbf{k}_t)$ to $H(f(\boldsymbol{\mu}_t, \boldsymbol{\tau}_t; \mathbf{k}_t), \mathbf{z}_t, \mathbf{L}_t, c_t) = 0$.

Financial Intermediary Aggregation Theory Under Homogeneity

In this section, we produce a **two-stage decision** for the financial intermediary. In the first stage, the firm solves for profit-maximizing factor demands and the profit-maximizing level of aggregate financial services produced. In the second stage, the revenue-maximizing vector of individual financial service quantities supplied is determined at fixed aggregate financial service quantity supplied.

To display that decomposition of the firm's profit-maximization decision, we start by defining the relevant revenue functions. The financial firm's revenue function is

$$W(\boldsymbol{\alpha}_t, \boldsymbol{\gamma}_{\mathbf{t}}, \widetilde{\boldsymbol{\pi}}_t, \boldsymbol{\sigma}_t, \boldsymbol{\zeta}_t; \mathbf{k}_{\mathbf{t}})$$
$$= \max_{\{\boldsymbol{\mu}_{\mathbf{t}}, \boldsymbol{\tau}_t\}} \{\boldsymbol{\mu}'_t \boldsymbol{\gamma}_t + \boldsymbol{\tau}'_t \tilde{\boldsymbol{\pi}}_{\mathbf{t}} + \boldsymbol{\zeta}'_t \boldsymbol{\sigma}_t : f(\boldsymbol{\mu}_t, \boldsymbol{\tau}_t; \mathbf{k}_t) = g(\boldsymbol{\alpha}_t)\}.$$

Since the decision is conditional on consumer choice of $\boldsymbol{\zeta}_t$, the financial firm's variable revenue function can be written as

$$R^*(\boldsymbol{\alpha}_t, \boldsymbol{\gamma}_{\mathbf{t}}, \widetilde{\boldsymbol{\pi}}_t; \mathbf{k}_{\mathbf{t}}) = \max_{\{\boldsymbol{\mu}_{\mathbf{t}}, \boldsymbol{\tau}_t\}} \{\boldsymbol{\mu}'_t \boldsymbol{\gamma}_t + \boldsymbol{\tau}'_t \tilde{\boldsymbol{\pi}}_{\mathbf{t}} : f(\boldsymbol{\mu}_t, \boldsymbol{\tau}_t; \mathbf{k}_t) = g(\boldsymbol{\alpha}_t)\},$$

where the firm selects $\boldsymbol{\alpha}_t$ to maximize variable profits

$$P_t = R^*(\boldsymbol{\alpha}_t, \boldsymbol{\gamma}_t, \widetilde{\boldsymbol{\pi}}_t; \mathbf{k}_t) - \boldsymbol{\alpha}'_t \boldsymbol{\beta}_t.$$

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However, by Shephard's (1970, p. 251) Proposition 83, it follows that there exists a linearly homogeneous output price aggregator function Γ such that

$$R^*(\boldsymbol{\alpha}_t, \boldsymbol{\gamma}_t, \tilde{\boldsymbol{\pi}}_t; \mathbf{k}_t) = \Gamma(\boldsymbol{\gamma}_t, \tilde{\boldsymbol{\pi}}_t) g(\boldsymbol{\alpha}_t).$$

Hence the financial firm's variable profits can alternatively be written as

$$P_t = \Gamma(\boldsymbol{\gamma}_t, \tilde{\boldsymbol{\pi}}_t) g(\boldsymbol{\alpha}_t) - \boldsymbol{\alpha}_t' \boldsymbol{\beta}_t.$$

Clearly, the **exact economic output quantity aggregate for the financial firm** is

$$M_t^s = f(\boldsymbol{\mu}_t^*, \boldsymbol{\tau}_t^*; \mathbf{k}_t),$$

when $(\boldsymbol{\mu}_t^*, \boldsymbol{\tau}_t^*)$ is the variable profit-maximizing vector of monetary assets and credit card transaction volumes produced. The corresponding variable output price aggregate is

$$\Gamma_t^s = \Gamma(\boldsymbol{\gamma}_t, \tilde{\boldsymbol{\pi}}_t).$$

With $g(\boldsymbol{\alpha}_t)$ set equal to 1.0, the variable output price aggregate is equal to

$$\Gamma(\boldsymbol{\gamma}_t, \tilde{\boldsymbol{\pi}}_t) = \max_{\{\boldsymbol{\mu}_t, \boldsymbol{\tau}_t\}} \{\boldsymbol{\mu}_t' \boldsymbol{\gamma}_t + \boldsymbol{\tau}_t' \tilde{\boldsymbol{\pi}}_\mathbf{t} : f(\boldsymbol{\mu}_t, \boldsymbol{\tau}_t; \mathbf{k_t}) = 1\},$$

which is the unit variable revenue function. The unit variable revenue function is the maximum variable revenue that can be acquired from the production of one unit of the output monetary aggregate, $M_t^s = f(\mu_t, \tau_t; \mathbf{k_t})$.

In stage-two, define the decision to be the selection of $(\boldsymbol{\mu}_{\boldsymbol{t}}^{*}, \boldsymbol{\tau}_{t}^{*})$ to minimize the aggregate factor requirement $f(\boldsymbol{\mu}_{t}, \boldsymbol{\tau}_{t}; \mathbf{k}_{t})$ subject to

$$\boldsymbol{\mu}_{t}'\boldsymbol{\gamma}_{t} + \boldsymbol{\tau}_{t}'\tilde{\boldsymbol{\pi}}_{\mathbf{t}} = \Gamma(\boldsymbol{\gamma}_{t}, \tilde{\boldsymbol{\pi}}_{t})g(\boldsymbol{\alpha}_{t}^{*}).$$

As a result, we obtain

$$M_t^s = \min_{\{\boldsymbol{\mu}_t, \boldsymbol{\tau}_t\}} \{ f(\boldsymbol{\mu}_t, \boldsymbol{\tau}_t; \mathbf{k}_t) : \boldsymbol{\mu}_t' \boldsymbol{\gamma}_t + \boldsymbol{\tau}_t' \tilde{\boldsymbol{\pi}}_t = \Gamma(\boldsymbol{\gamma}_t, \tilde{\boldsymbol{\pi}}_t) g(\boldsymbol{\alpha}_t^*) \},$$

while our earlier statement of the stage-two decision produces the equivalent result that

$$M_t^s \Gamma(\boldsymbol{\gamma}_t, \tilde{\boldsymbol{\pi}}_t) = \max_{\{\boldsymbol{\mu}_t, \boldsymbol{\tau}_t\}} \{ \boldsymbol{\mu}_t' \boldsymbol{\gamma}_t + \boldsymbol{\tau}_t' \tilde{\boldsymbol{\pi}}_t : f(\boldsymbol{\mu}_t, \boldsymbol{\tau}_t; \mathbf{k_t}) = g(\boldsymbol{\alpha}_t^*) \}.$$

Financial Intermediary Index Number Theory Under Homogeneity

Theorem 1. If $(\boldsymbol{\mu}_t^*, \boldsymbol{\tau}_t^*)$ is the financial intermediary's second-stage decision, then for every $t \in T_0$

$$d\log M_t^s/dt = \sum_i s_{it} d\log \mu_{it}^*/dt + \sum_j u_{jt} d\log \tau_{jt}^*/dt,$$

where $s_{it} = \mu_{it}^* \gamma_{it} / (\boldsymbol{\mu}_t^{*'} \boldsymbol{\gamma}_t + \boldsymbol{\tau}_t^{*'} \widetilde{\boldsymbol{\pi}}_t)$ and $u_{jt} = \tau_{jt}^* \tilde{\pi}_{jt} / (\boldsymbol{\mu}_t^{*'} \boldsymbol{\gamma}_t + \boldsymbol{\tau}_t^{*'} \widetilde{\boldsymbol{\pi}}_t)$.

Monetary Policy Transmission Mechanism

Macro models that seek to explain national income determination without including banks in the model implicitly assume there is no value added in banking, since national income data do not include adequate imputation of that value added. But private banks would not exist in equilibrium without producing value added.

Value Added from Financial Intermediation

Partition the financial intermediary's input vector $\boldsymbol{\alpha}_t$ so that $\boldsymbol{\alpha}_t = (\boldsymbol{\alpha}'_{1t}, \boldsymbol{\alpha}'_{2t})'$, where $\boldsymbol{\alpha}_{1t}$ is the quantities of primary inputs to the financial intermediary, and $\boldsymbol{\alpha}_{2t}$ is quantities of intermediate inputs. Partition the factor-price vector correspondingly so that $\boldsymbol{\beta}_t = (\boldsymbol{\beta}'_{1t}, \boldsymbol{\beta}'_{2t})'$. Then the financial intermediary's technology can be written as

$$M_t^s = g(\boldsymbol{\alpha}_{1t}, \boldsymbol{\alpha}_{2t}).$$

Let the firm's maximum variable profit level at given $\boldsymbol{\alpha}_{1t}$ be

$$V_t = V(\boldsymbol{\alpha}_{1t}, \boldsymbol{\beta}_{2t}, \boldsymbol{\gamma}_t, \tilde{\boldsymbol{\pi}}_t),$$

Sato (1975) calls

$$V_{t_0,t_1} = V(\boldsymbol{\alpha}_{1t_0}, \boldsymbol{\beta}_2^*, \boldsymbol{\gamma}^*, \tilde{\boldsymbol{\pi}}_t^*)/V(\boldsymbol{\alpha}_{1t_1}, \boldsymbol{\beta}_2^*, \boldsymbol{\gamma}^*, \tilde{\boldsymbol{\pi}}_t^*)$$

the **true index of real value added**, which depend upon the selection of the reference prices $(\boldsymbol{\beta}_2^*, \boldsymbol{\gamma}^*, \tilde{\boldsymbol{\pi}}^*)$.

The need to select the reference prices $(\boldsymbol{\beta}_2^*, \boldsymbol{\gamma}^*, \tilde{\boldsymbol{\pi}}^*)$ becomes unnecessary if and only if **if g is separable**, so that

$$M_t^s = G(\varphi(\boldsymbol{\alpha}_{1t}), \boldsymbol{\alpha}_{2t}).$$

In that case, V can be written

$$V_t = V_1(\boldsymbol{\alpha}_{1t}) V_2(\boldsymbol{\beta}_{2t}, \boldsymbol{\gamma}_t, \tilde{\boldsymbol{\pi}}_t).$$

So clearly

$$V_{t_0,t_1} = V_1(\boldsymbol{\alpha}_{1t_0}) / V_1(\boldsymbol{\alpha}_{1t_1}),$$

which does not depend on reference prices.

However, in this case $\varphi(\boldsymbol{\alpha}_{1t})$ is itself a category subproduction function, so we can more directly define the value added index to be

$$V^*_{t_0,t_1} = \varphi(\boldsymbol{\alpha}_{1t_0})/\varphi(\boldsymbol{\alpha}_{1t_1}).$$

If φ is translog, then the discrete Divisia index is exact for either V_{t_0,t_1} or $V^*_{t_0,t_1}$, so the discrete Divisia index provides a second-order approximation for $V^*_{t_0,t_1}$ or V_{t_0,t_1} for any φ. **In continuous time, the Divisia index is always exact for $\varphi(\boldsymbol{\alpha}_{1t})$, which is value added.** Alternatively the Divisia index over intermediate factors could be subtracted from the Divisia index over intermediate outputs to produce "double deflation" value added.

Future Research

- Econometrically model bank technology and include in a macro model to investigate the transmission mechanism.
- Investigate the role of shadow banking in transmission of policy.
  - Some shadow banking is relevant to the credit channel rather than to the monetary channel.
- Is the appearance of **risk averse behavior** by banks **incentive compatible**?

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- Incorporate the credit channel of policy through a separate weakly separable block, containing serviced bank loans. Credit card rotating balances, car loans, mortgage loans, and unsecured loans are among them, but not bonds, which are primary securities purchased by banks. Those bonds are not produced or serviced by banks.

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Results Available Soon

- **Remove currency** from the demand side credit-card-augmented Divisia aggregates to measure inside money.
  - The regulatory wedge produced by nonpayment of interest on required reserves is negligible during our sample period.
- Remove shadow banking assets from inside money, such as money market mutual funds, repurchase agreements, credit default swaps, and commercial paper, to acquire inside money produced by the banking system. Also remove Treasury bills.
- **Converting inside money into value added in banking** would require data on bank factors of production, separated into primary and secondary factors.

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Wikipedia Entry for "Monetary Base"

Monetary Base is also called:

- Base Money
- Money Base
- High-Powered Money
- Reserve Money
- **Outside Money**
- Central Bank Money
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It follows that simple sum M1 minus the monetary base could be called non-base money, low-powered money, or **inside money**.

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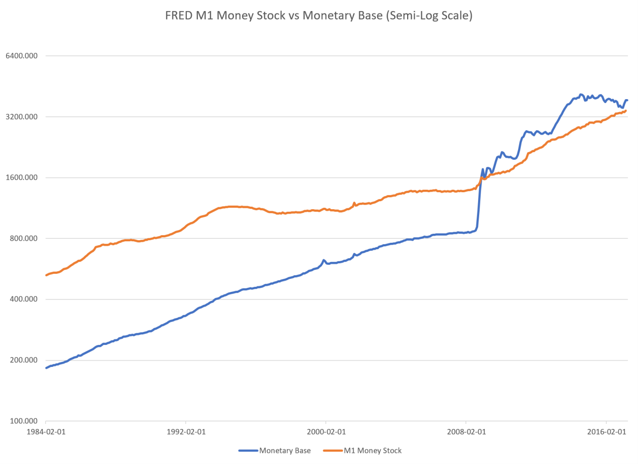
Outside Money

"Prior to the financial crisis, the monetary base consisted entirely of outside money."

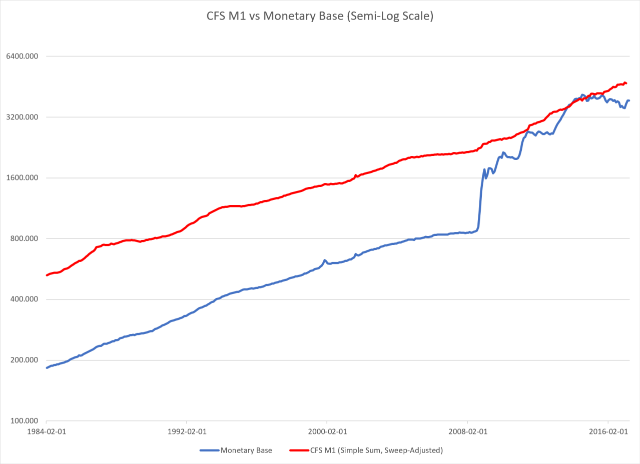
- Jeffrey Rogers Hummelon[2]

[2] Jeffrey Rogers Hummelon, "The Monetary Base and Total Reserves: Fed Confusions and Misreporting." Article published online in Alt-M (blog sponsored by the Cato Institute), November 7, 2015. https://www.alt-m.org/2015/11/07/monetary-base-total-reserves-fed-confusions-misreporting/

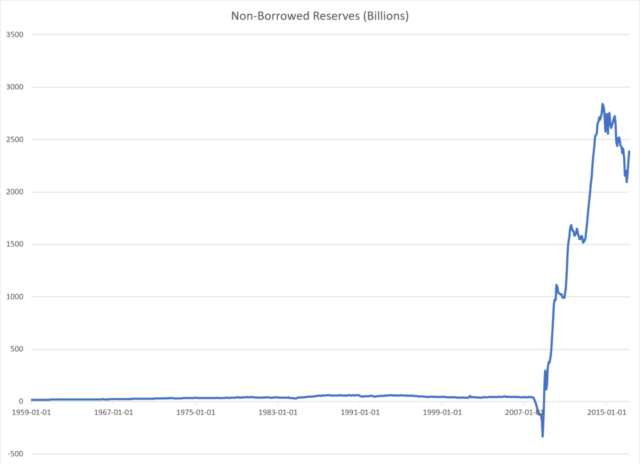
FRED Simple Sum M1 versus Monetary Base



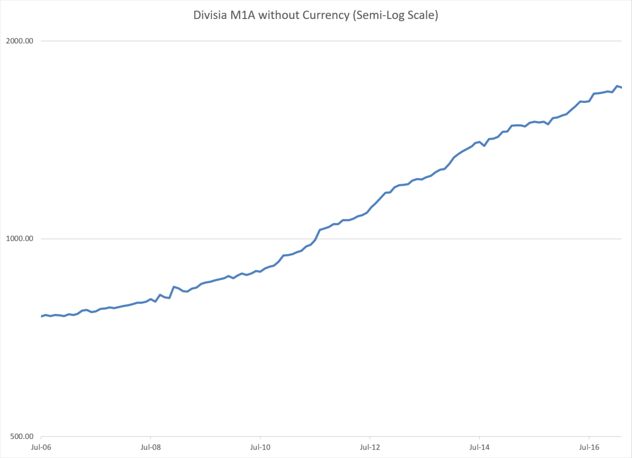
CFS Sweep-Adjusted M1 versus Monetary Base



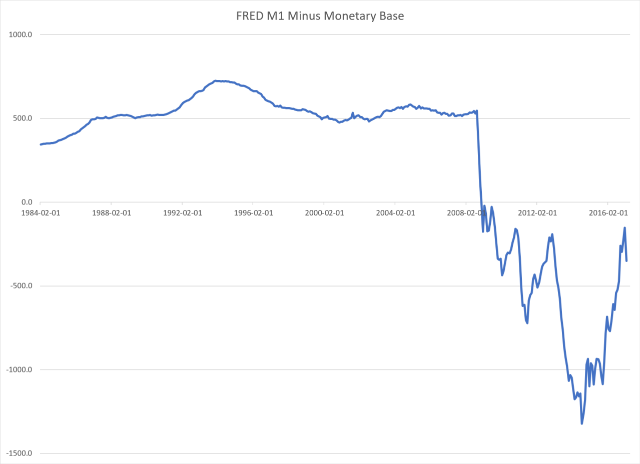
Non-Borrowed Reserves (FRED)



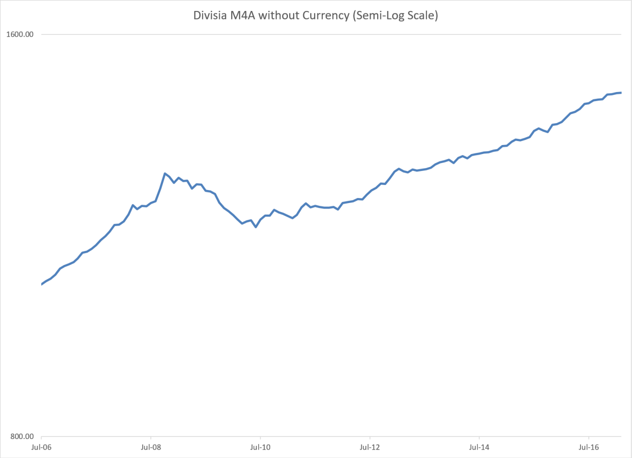
Augmented Divisia M1 with Currency Removed



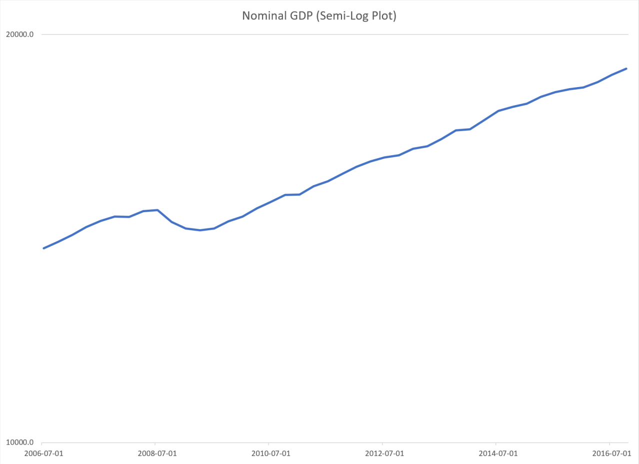
Inside Money Computed from FRED M1



Augmented Divisia M4 with Currency Removed



Nominal GDP


Nominal GDP (Semi-Log Plot)
20000.0
10000.0
2006-07-01
2008-07-01
2010-07-01
2012-07-01
2014-07-01
2016-07-01

Conclusion

- In this paper a monetary production model of financial firms is employed to investigate supply-side monetary aggregation augmented to include the credit card transactions services produced by those firms.
- Financial firms' outputs of demand deposits, time deposit services, and credit card transactions services can be aggregated to produce an output aggregate, which then enters an aggregate services supply function for the financial firm.
- When all outputs are separable from inputs, there exists a single output aggregate, and hence the use of a single output aggregate can be justified in the formulation and estimation of the financial firm's production technology.

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